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Dr. J. L. E. DREYER, PRESIDENT, in the Chair.

Francis John Bonham, 13 Dudley Road, Wimbledon, London, S.W. 19;

John Anthony Carroll, B.A., Assistant Director, Solar Physics Observatory, Research Fellow of Sidney Sussex College, Cambridge;

The Hon. Lionel Guest, 26 Connaught Square, London, W. 2, and Sea Lane, Ferring, Goring-by-Sea, Sussex;

Claude Haines Hall, Jr., Yale University Observatory, New Haven, Connecticut, U.S.A.;

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Bruno Philip Henry de Roeper, Squadron Leader R.A.F., No. 1 Flying Training School, Netheravon, Wilts; and

George A. Taylor, A.M.I.E. of Australia, 20 Loftus Street, Sydney, N.S.W., Australia,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

P. R. Chidambava Aiyar, Assistant at the Kodaikánal Observatory, S. India (proposed by T. Royds); and

Robert Barker, "Brendon," Crossbrook Street, Cheshunt, Herts (proposed by W. Goodacre).

Fifty-three presents were announced as having been received since the last meeting, including, amongst others:—

Uccle, Observatoire Royal de Belgique, Carte Photographique du Ciel, Zones $+33°$, $+35°$, $+37°$, $+39°$ (30 charts).

On the Masses, Luminosities, and Surface-Temperatures of the Stars.
By J. H. Jeans.

1. The present paper records an attempt to study the physical constitution of the stars without making the special assumptions which underlie Professor Eddington's investigations of the same problem. It is only after clearing the problem of these special assumptions that it becomes possible to say which of Professor Eddington's equations represent genuine properties of the stars and which merely represent the condition that a star shall conform to these special assumptions.

When the problem is treated in a sufficiently general way, the rather surprising properties which Professor Eddington attributes to the stars are found to disappear. In particular, it is found that there is no general relation between the masses and luminosities of the stars—the supposed "mass-luminosity" law disappears entirely as a theoretical law, so that a star of given mass can always adjust itself to radiate energy at any rate which happens to be required by the generation of energy in its interior. There is found to be a general relation between the mass, luminosity, and surface-temperature of a star, and this shows that the adjustment just spoken of can be made in only one way. In other words, the whole interior constitution of a star is uniquely determined by its mass and its rate of generation of energy.

The results of our investigation go some way towards rehabilitating the giant-dwarf theory of stellar evolution. A giant-dwarf sequence now appears as a possible, and indeed rather probable, life-history for a star. The bearing of our results on stellar evolution is discussed in § 18, to which some readers may prefer to turn at once.

2. The mathematical investigation which follows, although for the most part very different from that of Professor Eddington, overlaps it at certain points. I have, however, thought it best to make my investigation complete in itself, so as to secure logical continuity of exposition and also to show as clearly as possible what assumptions it involves.

The main point of difference between my work and that of Eddington is as follows:—In each of Eddington's investigations physical assumptions are introduced which result in the quantity which I denote by λ (the ratio of gas-pressure to pressure of radiation, Eddington's $\beta/(1-\beta)$) being constant throughout a star. In an actual star, even in the case most favourable for constancy, λ must vary as the fourth root of the temperature, and so will vary by a factor of at least ten between the outer regions and the centre. But λ may vary much more rapidly than this, and a variation of λ by a factor of 1000 between

regions in the same star can hardly be dismissed as impossible.* Our investigations differ mainly in that Eddington's is based on assumptions which make λ constant, while my investigation treats λ as a variable quantity whose value must be determined from point to point of the star by the analysis itself, and so avoids Eddington's special assumptions.

3. Let ρ, T, μ denote the density, temperature, and mean molecular weight at a point in the star's interior distant r from its centre. Let p, q denote the gas-pressure and pressure of radiation at the same point. Then

$$p = \frac{\mathrm{R}}{m\mu}\rho\mathrm{T} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$q = \tfrac{1}{3}a\mathrm{T}^4 \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where R is the universal gas-constant, m the mass of the hydrogen atom, and a the radiation-constant such that $a\mathrm{T}^4$ is the energy per unit volume of black-body radiation at temperature T. Numerically, in C.G.S. units,

$$\frac{\mathrm{R}}{m} = 8{\cdot}26 \times 10^7, \quad a = 7{\cdot}63 \times 10^{-15} \quad . \quad . \quad . \quad (3)$$

If γ is the gravitation-constant ($\gamma = 6{\cdot}66 \times 10^{-8}$), the dynamical equation of equilibrium for the star is

$$\frac{d}{dr}(p+q) = -\frac{\gamma\rho}{r^2}\int_0^r 4\pi\rho r^2 dr \quad . \quad . \quad . \quad (4)$$

If H is the outward flow of radiant energy per unit area, the equation of transfer of radiation is

$$\mathrm{H} = -\frac{16\sigma\mathrm{T}^3}{3c\rho}\frac{\partial\mathrm{T}}{\partial r} \quad . \quad . \quad . \quad . \quad (5)$$

where σ is Stefan's constant of radiation, equal to $\frac{1}{4}a\mathrm{C}$, and c is the opacity of the star at the point in question. If energy is generated at a rate $4\pi\epsilon$ per unit volume, the total flow of energy across a sphere of any radius r, surrounding the centre of the star, must be equal to the total generation of energy inside this sphere, so that

$$4\pi r^2\mathrm{H} = 4\pi\int_0^r 4\pi\epsilon\rho r^2 dr \quad . \quad . \quad . \quad . \quad (6)$$

Denoting the average value of ϵ inside a sphere of radius r by η, equation (6) may be put in the form

$$r^2\mathrm{H} = \eta\int_0^r 4\pi\rho r^2 dr \quad . \quad . \quad . \quad . \quad (7)$$

* At the actual boundary λ is necessarily zero, so that the factor of variation is strictly infinite for every star. But this hardly enters into the analysis, and the first value of λ that is important is that at depths several times deeper than we can see to.

and equation (5) assumes the form

$$\frac{4aC}{3c}T^3\frac{dT}{dr} = -\frac{\eta\rho}{r^2}\int_0^r 4\pi\rho r^2 dr \quad . \quad . \quad . \quad (8)$$

To these equations we must, in due course, add a further equation, namely, a relation between μ, T, and ρ giving the state of ionisation at any point in the star's interior in terms of the temperature and density at the point.

Equation (5) is accurate in the far interior of the star but begins to fail as the boundary is approached. I have, however, shown* that, near to a star's boundary, there is an approximate solution of the equations of radiative equilibrium, according to which

$$4\sigma T^4 = \frac{3}{2}H(1 + 2\int c\rho dr) \quad . \quad . \quad . \quad . \quad (9)$$

where the integral is taken from the point at which T and H are evaluated to the star's boundary. On differentiating with respect to r we find that equation (5) is true, to within the limits of this approximation, right up to the star's boundary, while on passing to the boundary equation (9) assumes the form (since $4\sigma = aC$)

$$\tfrac{1}{3}aT^4 = \frac{H}{2C} \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Although Milne† has shown how a still better approximation can be obtained, the foregoing is preferable for our present purpose. It introduces no error at all in the star's interior; at the surface it introduces an error of the order of 3 per cent. in the effective temperature, but this is insignificant in comparison with observational uncertainties. It has the great advantage that equation (5) holds true right up to the star's surface. We accordingly take equations (4) and (8) as the equations to be satisfied in the star's interior, and equation (10) as the boundary-condition to be satisfied at the boundary of the star, determined by the condition $\rho = 0$.

In his last paper Eddington gives a valuable summary of the results of recent theories of the dependence of the coefficient of opacity c on ρ, μ, and T.‡ All theories are in agreement that the main part of the variation of c is given by $c \propto \rho/\mu T^3$, but different theories supplement this by an additional temperature factor. According to both Eddington's theory of nuclear capture and the theory of Kramers there is a factor $T^{-\frac{1}{2}}$, while Kramers' theory calls for an additional factor of the form $(1 + h\nu/RT)$, which may in special cases give a total additional factor varying as rapidly as $T^{-\frac{7}{2}}$. Eddington assumes an additional factor $T^{-\frac{1}{2}}$. For the present we shall take this additional factor to be T^{-n}, and so put

$$c = \frac{\kappa\rho}{\mu T^3}T^{-n} \quad . \quad . \quad . \quad . \quad . \quad (11)$$

* *M.N.*, 1917 Nov., p. 35.
† *M.N.*, 1921 March, p. 361.
‡ *M.N.*, 1924 March, p. 309.

where κ is a constant. Equation (8) now assumes the form

$$\frac{4aC\mu}{3\kappa\rho}T^{6+n}\frac{dT}{dr} = -\frac{\eta\rho}{r^2}\int_0^r 4\pi\rho r^2 dr,$$

or, in terms of p and q defined by equations (1) and (2),

$$\frac{3CR}{a\kappa m}T^n\frac{q}{p}\frac{dq}{dr} = -\frac{\eta\rho}{r^2}\int_0^r 4\pi\rho r^2 dr \quad . \quad . \quad . \quad (12)$$

4. The same integral occurs in the two volume-equations (4) and (12). Eliminating it, we obtain

$$\frac{d}{dr}(p+q) = \frac{3CR\gamma}{a\kappa m}\frac{T^n}{\eta}\frac{q}{p}\frac{dq}{dr} \quad . \quad . \quad . \quad (13)$$

It is likely that ϵ, which measures generation of energy per unit volume, will be greater in the hot interior of a star than near its surface, so that η, the average value of ϵ inside a sphere of radius r, may decrease as we pass out to the cooler external regions. To represent this tendency, let us suppose that

$$\eta = \eta_0 T^l,$$

where η_0 and l are constants, l being small and positive. We can pass to the case of uniform generation of energy by putting $l = 0$. We can now put

$$\frac{T^n}{\eta} = \frac{T^{(n-l)}}{\eta_0} = \zeta q^s \quad . \quad . \quad . \quad . \quad (14)$$

where, since $q = \frac{1}{3}aT^4$, we have $4s = n - l$.

Equation (13) now assumes the form

$$\frac{d}{dr}(p+q) = K\frac{q^{1+s}}{p}\frac{dq}{dr} \quad . \quad . \quad . \quad . \quad (15)$$

where

$$K = \frac{3CR\gamma\zeta}{a\kappa m} \quad . \quad . \quad . \quad . \quad . \quad (16)$$

On putting $p = \lambda q$, so that λ denotes the ratio of gas-pressure to pressure of radiation, equation (14) becomes

$$\frac{d}{dr}[q(1+\lambda)] = K\frac{q^s}{\lambda}\frac{dq}{dr},$$

or, if λ is regarded as a function of q,

$$q\frac{\partial\lambda}{\partial q} = \frac{Kq^s}{\lambda} - (\lambda + 1) \quad . \quad . \quad . \quad . \quad (17)$$

5. In the special case of $s = 0$, the equation is immediately integrable and yields the general solution

$$(u+v)\log q + u\log(\lambda - u) + v\log(\lambda + v) = A \quad . \quad . \quad (18)$$

where A is a constant of integration and u, $-v$ are the roots of

$$K - \lambda(\lambda + 1) = 0 \quad . \quad . \quad . \quad . \quad (19)$$

If A is determined in terms of q_0, the value of q at the star's boundary (at which $\lambda = 0$), equation (18) assumes the form

$$u \log\left[\frac{q}{q_0}\left(1 - \frac{\lambda}{u}\right)\right] + v \log\left[\frac{q}{q_0}\left(1 + \frac{\lambda}{v}\right)\right] = 0 \quad . \quad . \quad (20)$$

The second term being necessarily positive, the first term must be negative, so that

$$\frac{q}{q_0}\left(1 - \frac{\lambda}{u}\right) < 1,$$

whence

$$\lambda = u\left(1 - \theta\frac{q_0}{q}\right) \quad . \quad . \quad . \quad . \quad (21)$$

where θ is a fraction less than unity. In the far interior of an actual star q_0/q is very small, so that λ approximates closely to the constant value u.

6. When s is different from zero, we write $Kq^s = x^2$, and equation (17) assumes the form

$$\tfrac{1}{2}sx\lambda\frac{\partial\lambda}{\partial x} = x^2 - \lambda(\lambda + 1) \quad . \quad . \quad . \quad . \quad (22)$$

It is found that the most general solution of this equation is obtained by assuming for λ^2 an expression of the form

$$\begin{aligned}\lambda^2 = x^2(&A + Bx^{-1} + Cx^{-2} + Dx^{-3} + \ldots) \\ &+ x^{-4/s}(\alpha + \beta x^{-1} + \gamma x^{-2} + \delta x^{-3} + \ldots) \\ &+ \text{terms in } x^{-8/s}, x^{-12/s}, \text{ etc.} \quad . \quad . \quad . \quad . \quad (23)\end{aligned}$$

By differentiation we obtain the value of $2\lambda\frac{d\lambda}{dx}$. If we write equation (22) in the form

$$\left[\tfrac{1}{2}sx\lambda\frac{d\lambda}{dx} + \lambda^2 - x^2\right]^2 = \lambda^2,$$

and substitute for the various terms from equation (23), we obtain, on equating coefficients of powers of x,

$$\begin{aligned}A &= \frac{2}{s+2}, \\ B^2(1 + \tfrac{1}{4}s)^2 &= A, \\ 2C(1 + \tfrac{1}{4}s) &= 1, \\ 2BD(1 + \tfrac{1}{4}s)(1 - \tfrac{1}{4}s) + C^2 &= C, \\ 2BE(1 + \tfrac{1}{4}s)(1 - \tfrac{1}{2}s) + 2CD(1 - \tfrac{1}{4}s) &= D, \text{ etc.} \quad . \quad . \quad (24)\end{aligned}$$

and also

$$\begin{aligned}-\tfrac{1}{2}sB\beta(1 + \tfrac{1}{4}s) &= \alpha, \\ -\tfrac{1}{2}s[C\beta + 2\gamma B(1 + \tfrac{1}{4}s)] &= \beta, \text{ etc.} \quad . \quad . \quad . \quad (25)\end{aligned}$$

Clearly a solution of equations (25) is obtained by taking

$$\alpha = \beta = \gamma = \ldots = 0,$$

so that a special solution is $\lambda^2 = \lambda_1^2$, where, from equations (24),

$$\lambda_1^2 = \frac{2}{s+2}x^2 \pm \frac{4}{s+4}\left(\frac{2}{s+2}\right)^{\frac{1}{2}}x + \frac{2}{s+4} \pm \frac{4(s+2)}{(4-s)(s+4)^2}\left(\frac{2}{s+2}\right)^{-\frac{1}{2}}x^{-1} + \ldots \quad (26)$$

and the general solution is

$$\lambda^2 = \lambda_1^2 + \lambda_2^2 \quad . \quad . \quad . \quad . \quad . \quad (27)$$

where, from equations (25),

$$\lambda_2^2 = ax^{-4/s}\left[1 \mp \frac{2}{s}\left(\frac{s+2}{2}\right)^{\frac{1}{2}}x^{-1} + \ldots\right] \quad . \quad . \quad (28)$$

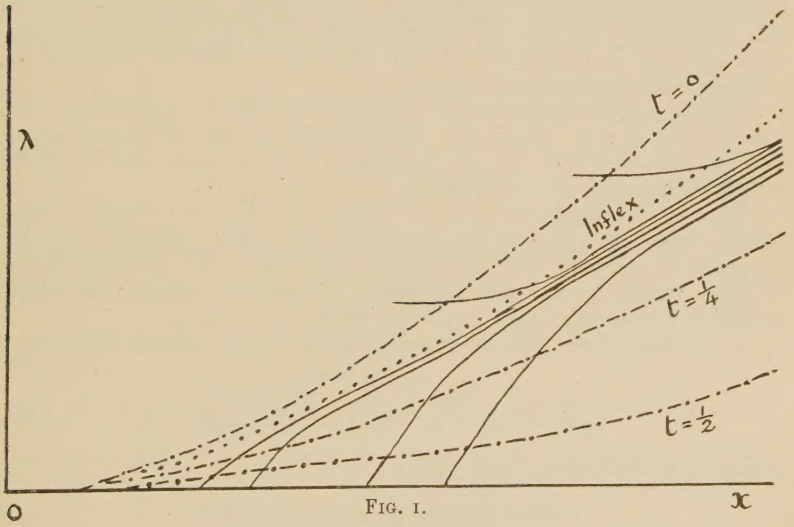

FIG. 1.

Since the original equation (22) was of the first order and second degree, this clearly represents the most general solution.

7. Equation (22) and its solution (27) are easily interpreted graphically. Let fig. 1 represent the plane of λx and let the continuous curves represent the family of curves specified by equation (22). The locus of points on these curves at which the tangents are parallel to the axis of λ is obtained by putting $d\lambda/dx = 0$ in equation (22), and is accordingly the hyperbola

$$x^2 - \lambda(\lambda + 1) = 0 \quad . \quad . \quad . \quad . \quad . \quad (29)$$

In the same way the locus of points at which the tangents to the curves make an angle $\tan^{-1} t$ with the axis of x is found to be the system of hyperbolas

$$x^2 - \tfrac{1}{2}stx\lambda - \lambda(\lambda + 1) = 0.$$

These are the chain curves marked $t = 0$, $t = \frac{1}{4}$, $t = \frac{1}{2}$, etc., in fig. 1.

Finally, the locus of points at which the curves have points of inflexion, given by $d^2\lambda/dx^2 = 0$, is found to be

$$\tfrac{1}{2}s\lambda^2[x^2+\lambda(\lambda+1)] = (x^2+\lambda^2)[x^2-\lambda(\lambda+1)],$$

giving the dotted curve marked Inflex. in fig. 1. The curves are now easily drawn in, and are found to lie as in fig. 1.

The curves which meet the positive axis of x represent physically possible configurations for stars in free space; the remainder could represent masses of gas in a spherical vessel.

8. Regarding the value of λ_1^2 given by equation (26) as a standard solution for λ^2, the divergence of any other solution is λ_2^2 given by equation (28). This contains the factor $x^{-4/s}$, of which the index is large when s is small. Thus for small values of s any initial divergence from the standard solution decreases very rapidly as x increases. By the time that x is equal to several times its initial value, the divergence λ_2^2 may be neglected, so that the solution may be supposed to coincide with the standard solution.

This is exactly similar to the situation we found in discussing the case of $s=0$ in § 5. There the standard solution was $\lambda = u$, and equation (21) showed how all the solutions closed in upon the standard solution when q/q_0 became large.

Thus in the interior of a star, in regions in which x is large in comparison with its value at the boundary, the actual solution may be supposed to coincide with the standard solution. The multiplicity of solutions merely has the effect of making it possible to satisfy a boundary condition which may assign any value whatever to x or q.

9. Writing y^2 for Ax^2, the standard solution (26) assumes the form

$$\lambda^2 = y^2 \pm \frac{y}{1+\frac{1}{4}s} + \frac{1}{2(1+\frac{1}{4}s)} \pm \frac{1}{8y}\frac{(1+\frac{1}{2}s)}{(1-\frac{1}{4}s)(1+\frac{1}{4}s)^2} - \quad . \quad . \quad . \quad . \quad (30)$$

When $s=0$, this reduces to

$$\lambda^2 = y^2 + \tfrac{1}{2} \pm [y^2+\tfrac{1}{4}]^{\frac{1}{2}} = \{[y+\tfrac{1}{4}]^{\frac{1}{2}} \pm \tfrac{1}{2}\}^2 \quad . \quad . \quad (31)$$

whence, taking the lower sign, we readily obtain

$$\lambda(\lambda+1) = y^2 = Ax^2 \quad . \quad . \quad . \quad . \quad (32)$$

When $s=0$, $Ax^2 = K$, so that our equation (32) becomes identical with our previous equation (19), and our standard solution reduces to the previous standard solution $\lambda = u$.

The transformation given in equation (31) shows that the series (30) is convergent in the special case of $s=0$, so long as $y^2 > \frac{1}{4}$, or so long as $\lambda > 0{\cdot}207$. In Eddington's notation $[\lambda = \beta/(1-\beta)]$ the corresponding condition is $1-\beta < 0{\cdot}828$, which, when $s=0$, is satisfied for all stars of mass less than 124 times the mass of the Sun. When s is different from zero, the limits of convergence will be somewhat different, but as we shall only be concerned with small values of s, we may assume the series to be convergent for all stars of reasonable mass.

10. For a star of small mass, radiation-pressure is insignificant in comparison with gas-pressure, so that λ is large. Neglecting all terms except those of highest degree in λ, we obtain from § 6 the approximate solution

$$\lambda^2 = \frac{2}{s+2}x^2 + ax^{-4/s};$$

or, restoring the value of x^2,

$$\lambda^2 = \frac{2\mathrm{K}}{s+2}\left(q^s - \frac{{q_0}^{s+2}}{q^2}\right) \quad . \quad . \quad . \quad . \quad (33)$$

This solution could have been obtained directly from equation (22) on replacing $\lambda(\lambda+1)$ by λ^2 and integrating. The constant q_0 may not legitimately be identified with the value of q at the boundary, since our solution must not be extended into regions in which λ is small. It will, however, be comparable with this value, so that in the star's interior the solution may be replaced by

$$\lambda^2 = \frac{2\mathrm{K}}{s+2}q^s \quad . \quad . \quad . \quad . \quad . \quad (34)$$

whence

$$p = q\lambda = \left(\frac{2\mathrm{K}}{s+2}\right)^{\frac{1}{2}}(\tfrac{1}{3}a\mathrm{T}^4)^{\frac{1}{2}s+1} \quad . \quad . \quad . \quad (35)$$

Eliminating T between this and the gas-equation (1), we find

$$p = \delta\left(\frac{\rho}{\mu}\right)^{\frac{2s+4}{2s+3}} \quad . \quad . \quad . \quad . \quad (36)$$

where

$$\delta^{2s+3} = \left(\frac{\mathrm{R}}{m}\right)^{2s+4}\left(\frac{2\mathrm{K}}{s+2}\right)^{-\frac{1}{2}}(\tfrac{1}{3}a)^{-\frac{1}{2}(s+2)}.$$

11. For a star of very great mass, λ is small. On passing to the limit in which λ is negligible, we find, as the solution of equation (22),

$$\lambda = x^2 = \mathrm{K}q^s,$$

whence we obtain, by the method just employed,

$$p = \theta\left(\frac{\rho}{\mu}\right)^{\frac{s+4}{4s+3}} \quad . \quad . \quad . \quad . \quad . \quad (37)$$

where

$$\theta^{4s+3} = \left(\frac{\mathrm{R}}{m}\right)^{4s+4}\mathrm{K}^{-1}(\tfrac{1}{3}a)^{-(s+1)}.$$

Also

$$p = \lambda q = \mathrm{K}q^{s+1},$$

so that

$$q = \left(\frac{p}{\mathrm{K}}\right)^{\frac{1}{s+1}} = \left(\frac{\theta}{\mathrm{K}}\right)^{\frac{1}{s+1}}\left(\frac{\rho}{\mu}\right)^{\frac{4}{4s+3}} \quad . \quad . \quad . \quad (38)$$

12. In each of the limiting cases just considered (λ very great and

λ very small) the sum $p+q$ reduces to one or other of its two terms, so that we have in each case a relation of the type

$$p+q=\mathrm{U}\left(\frac{\rho}{\mu}\right).$$

The value of μ will depend on both the density and the temperature, but since we are treating the temperature as a function of the density, it will be enough to regard μ as a function of the density only. Let us assume that

$$\frac{\rho}{\mu}\propto\rho^{\phi} \quad . \quad . \quad . \quad . \quad . \quad (39)$$

then the value of $p+q$ assumes the form

$$p+q=\mathrm{S}\rho^{n} \quad . \quad . \quad . \quad . \quad . \quad (40)$$

where $n=k\phi$.

The dynamical equation of equilibrium, namely,

$$\frac{d}{dr}(p+q)=-\frac{\gamma\rho}{r^2}\int_0^r 4\pi\rho r^2 dr \quad . \quad . \quad . \quad (41)$$

cleared of the integral, becomes, on substituting from equation (40)

$$\frac{1}{r^2}\frac{d}{dr}\left[\frac{n\mathrm{S}}{4\pi\gamma}r^2\rho^{n-2}\frac{d\rho}{dr}\right]+\rho=0.$$

If we put

$$r^2=\frac{n\mathrm{S}}{(n-1)4\pi\gamma}\mathrm{R}^2\,;\ \rho^{n-1}=\sigma \quad . \quad . \quad . \quad (42)$$

this becomes

$$\frac{1}{\mathrm{R}^2}\frac{d}{d\mathrm{R}}\left(\mathrm{R}^2\frac{d\sigma}{d\mathrm{R}}\right)+\sigma^{\frac{1}{n-1}}=0 \quad . \quad . \quad . \quad (43)$$

which is the standard form discussed by Emden.* The only solutions of astrophysical interest are those for which $d\sigma/d\mathrm{R}$ vanishes when $\mathrm{R}=0$. If R_1, σ_1 are the values of R and σ in any such solution, the most general solution will be

$$\sigma=f\sigma_1\,;\ \mathrm{R}=f^{\frac{n-2}{2n-2}}\mathrm{R}_1 \quad . \quad . \quad . \quad . \quad (44)$$

where f is a constant which may be adjusted to suit the particular problem in hand. We shall assume R_1, σ_1 to refer to the particular solution tabulated by Emden.

The mass M of star is found from equations (41) and (40) to be

$$\mathrm{M}=-\frac{r^2}{\gamma\rho}\frac{d}{dr}(p+q)=-\frac{r^2\mathrm{S}n}{\gamma}\rho^{n-2}\frac{d\rho}{dr},$$

which, transformed by relations (42) and (44), becomes

$$\mathrm{M}=-4\pi\left[\frac{n\mathrm{S}}{(n-1)4\pi\gamma}\right]^{\frac{3}{2}}f^{\frac{3n-4}{2n-2}}\mathrm{R}_1{}^2\frac{d\sigma_1}{d\mathrm{R}_1} \quad . \quad . \quad . \quad (45)$$

where $\mathrm{R}_1{}^2\,d\sigma_1/d\mathrm{R}_1$ is evaluated at the boundary of the star.

* *Gaskugeln*, pp. 61 *et ff.*

The total rate of emission of energy E, and the rate of emission per unit area H, are connected with M by the equations (*cf.* equations (6) and (7))

$$E = 4\pi r^2 H = 4\pi\eta M \quad . \quad . \quad . \quad . \quad (46)$$

while from equation (10)

$$H = 2Cq_0 \quad . \quad . \quad . \quad . \quad . \quad (47)$$

From equations (14)

$$\zeta = \frac{1}{(\frac{1}{3}a)^s \eta_0},$$

so that equation (16) gives

$$K = \frac{CR\gamma}{(\frac{1}{3}a)^{s+1} km\eta_0}.$$

Thus when η_0 is fixed, K and therefore also S is fixed, so that equation (45) gives M in terms of f. In virtue of equations (46) and (47), equation (45) may be regarded either as expressing a relation between η, M, and f, or between E, M, and f. Whatever the values of E and M, a value of f can be found to satisfy the dynamical equation of equilibrium (45), and when this has been found equations (44) and (47) will give the values of r and q_0.

Thus mathematical analysis brings us back to the common-sense view that a star of given mass M, generating energy at a given rate E, can always adjust its radius and surface-temperature so as to be in equilibrium. If, as in fig. 2,* a diagram is drawn in which E and M are taken for ordinate and abscissa, every point in the positive quadrant of this diagram represents a possible configuration for a star. There is no limitation to a single curve or narrow band across this diagram such as is exhibited in Eddington's figure.† If the stars are found in actual fact to lie on a narrow band, there must be extraneous reasons for the absence of stars outside this band; they are in no way driven from these exterior regions by the laws of physics.

13. Returning to the case of a star for which λ is large, we have already found in § 11 that

$$S = \delta = \left(\frac{R}{m}\right)^{\frac{2s+4}{2s+3}} \left(\frac{2K}{s+2}\right)^{-\frac{1}{2(2s+3)}} \left(\frac{1}{3}a\right)^{-\frac{s+2}{2(2s+3)}}$$

$$n = k\phi = \frac{2s+4}{2s+3}\phi.$$

Equation (45), written in logarithmic form to save printing, now assumes the form

$$\log M = \frac{3}{4(2s+3)} \log \eta_0 + \frac{3n-4}{2n-2} \log f + \text{cons.} \quad . \quad . \quad (48)$$

where the constant term on the right contains only absolute constants of nature.

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* See p. 210.

† *M.N.*, 84 (1924), 311.

In the same way, for a star for which λ is very small,

$$S = \left(\frac{\theta}{K}\right)^{\frac{1}{s+1}}, \qquad n = \frac{4\phi}{4s+3},$$

and equation (45) assumes the form

$$\log M = \frac{6}{4s+3}\log\eta_0 + \frac{3n-4}{2n-2}\log f + \text{cons.} \quad . \quad . \quad (49)$$

14. It will be remembered that s has stood for $\frac{1}{4}(n-l)$, where T^l is the temperature factor in η and T^{-n} is the additional temperature factor in c, the coefficient of opacity, defined by equation (11). It now appears that n and l do not enter separately into the formulæ so far obtained, but only through the combination $n-l$ or s.

For the present let us consider the simple case in which $l=0$, so that η has the constant value η_0. Then

$$E = 4\pi\eta_0 M,$$

so that

$$\log\eta_0 = \log E - \log M + \text{cons.}$$

and equations (48) and (49) assume the forms

(λ large)

$$(2s+3{\cdot}75)\log M = \tfrac{3}{4}\log E + (2s+3)\left(\frac{3n-4}{2n-2}\right)\log f \quad . \quad (50)$$

(λ small)

$$(4s+9)\log M = 6\log E + (4s+3)\left(\frac{3n-4}{2n-2}\right)\log f. \quad . \quad (51)$$

At the surface of the star, from equations (47), (46), (44), and (42),

$$\tfrac{1}{3}aT^4 = q_0 = \frac{H}{2C} = \frac{E}{8\pi Cr^2} = \frac{(n-1)\gamma}{2CnSR_1^2}Ef^{-\frac{n-2}{n-1}},$$

so that

$$4\log T = \log E - \log S - \left(\frac{n-2}{n-1}\right)\log f + \text{cons.},$$

and from equation (45)

$$\log M = \tfrac{3}{2}\log S + \left(\frac{3n-4}{2n-2}\right)\log f + \text{cons.}$$

Eliminating S from these last two equations, we have

$$4\log T + \tfrac{2}{3}\log M = \log E + \frac{2}{3(n-1)}\log f + \text{cons.}. \quad . \quad (52)$$

If we eliminate f between this equation and equation (50), we obtain (for a star for which λ is large)

$$3Q\log T + (\tfrac{1}{2}Q - 2s - 3{\cdot}75)\log M + \tfrac{3}{4}(1-Q)\log E = \text{cons.} \quad . \quad (53)$$

where

$$Q = (2s+3)(3n-4) = 12(\phi-1) + (6\phi-8)s \quad . \quad . \quad (54)$$

The emission E of a star is connected with its absolute magnitude m by the relation

$$\log E = -0{\cdot}4m + \text{cons.},$$

so that equation (53) assumes the form

$$c_1 \log T + c_2 \log M + c_3 m = \text{cons.} \quad . \quad . \quad . \quad (55)$$

where

$$\left.\begin{aligned} c_1 &= 3Q && = 36(\phi - 1) + (18\phi - 24)s \\ c_2 &= -(2s + 3{\cdot}75) + \tfrac{1}{2}Q && = -(2s + 3{\cdot}75) + 6(\phi - 1) + (3\phi - 4)s \\ c_3 &= 0{\cdot}3(Q - 1) && = 0{\cdot}3[-1 + 12(\phi - 1) + (6\phi - 8)s] \end{aligned}\right\} \quad (56)$$

15. If the values of T, m, and M were known for any three stars with absolute precision, it would be possible to use the data for these three stars to evaluate the ratio of $c_1 : c_2 : c_3$, and so to determine the values of ϕ and s. All the coefficients in equation (55) would then be known, and if our theory were perfect and complete, this equation ought to be satisfied by every gaseous star in the universe.

Instead of evaluating c_1, c_2, and c_3 from three selected stars, it seemed better to take an average valuation from six stars for which T, m, and M may be regarded as tolerably well known, namely, the Sun, the two components of α Centauri, the two components of Capella and Sirius. The observational data for these (which are taken from Eddington's paper) are given in the first five columns of Table I. The best fit is obtained by giving to equation (55) the form (very approximately)

$$\tfrac{5}{3}\log T + 8{\cdot}75 \log M + m = 11{\cdot}17 \quad . \quad . \quad . \quad (57)$$

The sixth column gives the value of the left-hand member of this equation calculated from the data given in the table; the last column, headed $\Delta M/M$, gives the proportionate amount by which M would have to be reduced to reduce the left-hand member of equation (57) to precisely 11·17.

TABLE I.

Star.	T.	M ($\odot = 1$).	log T.	log M.	m.	Left Hand of (57).	$\Delta M/M$ (O − C).
Sun	5860	1·00	3·768	0·000	4·89	11·17	0·000
α Centauri b	5000	1·14	3·699	0·057	4·53	11·19	+0·005
α Centauri f	3700	0·97	3·568	9·987	5·24	11·07	−0·027
Capella b	5200	4·18	3·716	0·621	−0·36	11·27	+0·027
Capella f	7400	3·32	3·869	0·521	0·22	11·23	+0·016
Sirius	10500	2·45	4·021	0·389	0·97	11·08	−0·024

The errors in the last column are almost within the errors of observation.

If, however, we identify equation (55) with equation (57), and solve for s and ϕ, we obtain the impossible values

$$\phi = 0{\cdot}95 \quad \text{and} \quad s = -0{\cdot}35.$$

On physical grounds it is clear that ϕ must be greater than unity, and s is almost certainly positive in actual fact. Reasonable values to take would probably be

$$\phi = 1{\cdot}025 \quad \text{and} \quad s = +0{\cdot}35 \quad . \quad . \quad . \quad (58)$$

When substituted in equations (56) these give

$$c_1 = -0{\cdot}75, \qquad c_2 = -4{\cdot}47, \qquad c_3 = -0{\cdot}375,$$

and equation (55) assumes the form

$$(\lambda \text{ great}) \qquad m + 2 \log \mathrm{T} + 11{\cdot}92 \log \mathrm{M} = \text{cons.} \quad . \quad . \quad . \quad (59)$$

For the stars already discussed the values assumed by the left-hand member of this equation are as follows :—

Star.	Mass ($\odot = 1$).	Left Hand of (58)	ΔM/M.
Sun	1·00	12·43	0·000
α Centauri *b*	1·14	12·61	+0·935
α Centauri *f*	0·97	12·22	−0·039
Capella *b*	4·18	14·47	+0·479
Capella *f*	3·32	14·17	+0·396
Sirius	2·45	13·65	+0·264

Clearly there is a systematic divergence with increasing mass. Probably a large part of the error can be attributed to λ being too large, in the more massive stars, for $1/\lambda$ to be neglected. The question seems to call for a discussion in which more than the first term in the solution (26) is taken into account, but such a discussion is not given in the present paper.

16. Using the values (58) for ϕ and s, the value of n for a star for which λ is very small is (*cf.* equations (38), (39), and (40))

$$n = \frac{4}{4s+3}\phi = 0{\cdot}932,$$

so that equations (51) and (52) assume the forms

$$10{\cdot}4 \log \mathrm{M} = 6 \log \mathrm{E} - \frac{5{\cdot}3}{2n-2} \log f + \text{cons.},$$

$$4 \log \mathrm{T} + \tfrac{2}{3} \log \mathrm{M} = \log \mathrm{E} + \frac{1{\cdot}33}{2n-2} \log f + \text{cons.},$$

whence by elimination of f we obtain, very approximately,

$$(\lambda \text{ small}) \qquad 16 \log \mathrm{T} + 13 \log \mathrm{M} = 10 \log \mathrm{E} + \text{cons.} \quad . \quad . \quad (60)$$

or, in terms of the absolute magnitude m,

$$(\lambda \text{ small}) \qquad m + 4 \log \mathrm{T} + 3{\cdot}25 \log \mathrm{M} = \text{cons.} \quad . \quad . \quad . \quad (61)$$

17. Equations (59) and (61) give the relations between m, T, and M for stars for which λ is very large and very small respectively. In some respects the results are in close agreement with those obtained by Eddington. For a star of very small mass (λ great) our equation (59) shows that when the temperature is kept constant

$$E \propto M^{4\cdot77},$$

while Eddington's theory gives $E \propto M^{4\cdot4}$. The difference in index is due merely to Eddington having taken the values $\phi = 1$ and $s = 0$ as an approximation. For stars of very great mass our equation (60) shows that when T is kept constant

$$E \propto M^{1\cdot3},$$

while Eddington's theory gives $E \propto M^{1\cdot4}$, the difference in the two indices having the same origin as before. As regards temperature-variation, Eddington's suggested temperature-correction $-2\log_{10}(T/5200)$ to be added to m* exactly represents the term $2 \log T$ in our equation (59). This, however, appears to be only a coincidence; it does not represent the term $4 \log T$ in equation (61).

18. Although our theory adds very little in the way of numerical results to what has already been given by Eddington, it leads to a very different interpretation of these results. Eddington's theory, based originally on the assumption $s = 0$, predicted a hard-and-fast relation between the rate of emission of radiation E of a star and its mass M—the supposed "mass-luminosity" law. This relation can be represented by a single curve in a diagram in which E and M are taken as co-ordinates; and the observed values of E and M for actual stars ought, according to Eddington, to lie on, or close to, this curve.

Our wider theory has shown that as soon as s differs even infinitesimally from zero, there is not one such curve in the E, M plane, but an infinite number. These various curves correspond to different values of the surface-temperature T, and they fill the whole plane, so that all values are possible for E and M separately. The only part of the plane which is ruled out is that in which the star would not be in the gaseous state. When s is made zero, these curves all coincide in a single curve (Eddington's theoretical "mass-luminosity" curve), but as soon as s differs again in the tiniest degree from zero, the curves spread out and immediately fill the whole plane. No one of these curves can claim to express a definite "mass-luminosity" law, and the curve along which they all fell when s was zero has no longer any special significance. In fact, the mass-luminosity law disappears entirely.

Equations (60) and (61) give the limiting forms of the curves in the regions of M small and M large respectively, and from these we can construct the whole diagram. The curves are found to lie as in fig. 2. As a star evolves, its representative point is free to move over the whole of this plane, and different hypotheses as to the source and mechanism of its generation of radiation will give different types of evolutionary curves.

* *Loc. cit.*, p. 310.

Eddington's curve would be approximately represented by any one of the curves of constant temperature. There seems to be no reason why a star should keep its surface-temperature constant throughout its evolution, so that evolution along any one of these curves, or in strict accordance with the supposed "mass-luminosity" law, would seem to be entirely improbable.

On the other hand, one of the curves in our diagram, say ABCO, will correspond to the surface-temperature T_1 which divides dark stars from visible ones. Thus on plotting observed values of E against observed values of M, the representative points will necessarily all fall

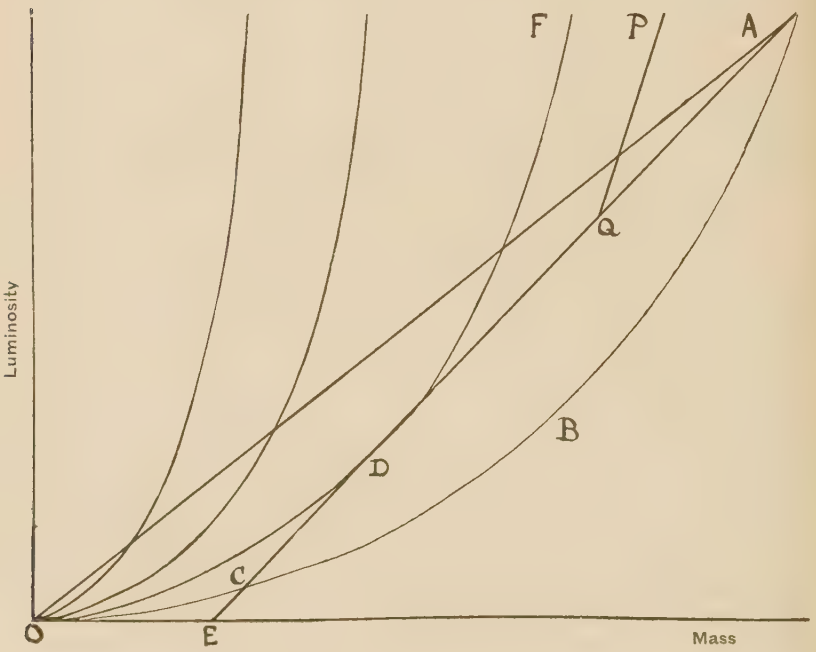


FIG. 2.

above the curve ABCO. We may, if we like, fix a limit in the upper direction also by imagining a surface-temperature T_2 such that when a star's surface exceeds this temperature it radiates energy with such extreme rapidity that its surface-temperature falls almost immediately below T_2. If the curve for T_2 is the curve FDO in our diagram, then all observed stars will fall between the two curves FDO and ABO. This agrees approximately with the facts of observation as shown in Eddington's diagram. Or, alternatively, we might suppose an upper limit fixed by a straight line such as AO, expressing that a star could never generate energy at more than a certain rate per unit mass.

Such explanations are clearly very imperfect, but we can only go further by introducing detailed theories of the origin of stellar radiation.

If a star remained of constant mass, its evolutionary course would be represented by a vertical line in the diagram. We should have red stars crossing the limiting curve ABEO throughout its length, and this would be very difficult to reconcile with the observed separation of red stars into giants and dwarfs.

If a star radiates on account of wastage of mass, E/M will measure the rate of breakdown. The evolutionary path of a star for which E/M remained constant would be a straight line such as AO through the origin. The surface-temperature of such a star would continually increase as its evolution progressed, contrary of course to what appears to happen in actual stars.

If a star were formed of a mixture of two kinds of matter, one of which broke down at a constant rate and the other not at all, its evolutionary path would be a straight line such as ADCE, not passing through the origin. Such a star would first become visible at A, would steadily increase in surface-temperature until this attained a maximum at D, after which it would decrease until the star became invisible at C. It is easily verified that the density would steadily increase throughout the whole of the star's life, except that there might be a falling off in the latest stages of all of the star's existence when the mass and luminosity both became quite small.

On the lines of this last example it seems possible to explain perfectly all the facts usually associated with the giant-dwarf hypothesis. Such an explanation does not, of course, limit us to supposing a star to consist only of two kinds of matter. Let us suppose it formed initially of masses M_1, M_2, M_3 . . . of different kinds of matter, breaking down with different moduli of decay κ_1, κ_2, κ_3 . . . . After any time t the mass of the star will be given by

$$M = M_1 e^{-\kappa_1 t} + M_2 e^{-\kappa_2 t} + M_3 e^{-\kappa_3 t} + \ . \ . \ . \ ,$$

and the rate of emission of radiation by

$$E = -C^2 \frac{dM}{dt} = C^2(\kappa_1 M_1 e^{-\kappa_1 t} + \kappa_2 M_2 e^{-\kappa_2 t} + \kappa_3 M_3 e^{-\kappa_3 t} + \ . \ . \ .).$$

On eliminating t we get a relation between E and M which will be represented by a broken line of the type PQEO in fig. 2, when κ_1, κ_2, κ_3 . . . are sufficiently unequal, and by a similar line with the corners rounded off when κ_1, κ_2, κ_3 . . . are less unequal. According to this explanation, there seems to be no reason in theory why the evolution of a star should not be through a whole succession of alternations of risings and fallings of the surface-temperature, what we call the giant and dwarf states forming only the two last members of a long series.

Effective Temperatures of Sixty-Four Stars. By Professor R. A. Sampson, F.R.S. (Plates 9, 10, 11.)

The following paper is the continuation of one published in *Monthly Notices*, 83, 1923 January, "On the Estimation of the Continuous Spectrum of Stars," in which the Koch microphotometer was used to reproduce and measure the density of a spectrogram throughout its length, and theory was then applied to eliminate the various irrelevant factors entering these measures. Thus was derived, relative to Polaris, taken as standard, the radiation curve of the star and its effective temperature. Reference may be made to that paper for details which need not be recalled here, except where there is some variation. It may, however, be said that the spectra are taken with an objective prism of 12° angle in front of a 6-inch photovisual object glass, giving a sharp focus from (B) to (K), with a dispersion over all of 1 mm. = 121 A. This spectrum is reproduced automatically as a graph—termed the *photogram*—by the Koch photometer, with a magnification of 10·5 in w.l., a function of the density being shown as the ordinate. Lines and bands show themselves as serrations of greater or less depth. Some examples are annexed, and others will be found in the paper referred to. Two major points were indicated in that paper (p. 204) as requiring special attention. First, the range of densities employed was rather low. Either the measures of the weaker part of the spectrum might become uncertain, or the summit of the curve become featureless. To cure this, spectra of greater density are now employed throughout, and, when the curve approaches the summit, the scale is changed by interposing a screen of ascertained density in the path of the beam which passes to the balancing cell, and the trace is thus brought back again to the neighbourhood of zero. The result is a broken curve, less convincing to the eye than the continuous curve, but for measurement possessing the additional advantage that each break serves to check any change of zero-density in the plate due to a change in the background of "clear plate," which must otherwise, in default of a test, be assumed uniform. The second point related to the treatment of photographic theory; adopting the current theory, it was found that the so-called "development constant" required to be adopted without direct and reliable determination from the material.

In the period that has elapsed a great deal of time and study has been devoted to this question. The experimental work has been in the hands of Mr. E. A. Baker, and is the subject of a separate publication, in detail, under his name.* The results only will be quoted here to elucidate and justify the method adopted. It may be said, however, that it is now claimed that the method gives a correct measure of the relative intensity of two celestial sources of radiation, for any wavelength, without the admixture of any empiric feature. The faults that remain are such as one would expect to remove (i) by more rigorous standardising of the material and thereby clearing the irregularities which some of the comparison curves show at different points of their

* *Proc. Roy. Soc. Edin.*, xlv.

length, and (ii) by a thorough study of atmospheric absorption, which is as necessary here, in order to avoid anomalous results, as is the study of atmospheric refraction for positional astronomy. In other words, what is required further is repetition, in order to improve the technique.

Photographic Theory.

Let a definite batch of plates be treated in a definite way, as by standard washing and drying before exposure, and after exposure, by developing with a standard developer for standard time at a standard temperature; then the density of the image for any part of the spectrum will depend only upon the intensity of the incident light and the exposure-time as variables, the associated constants for that batch of plates and treatment depending upon the wave-length only.

Adopt the usual notation

$$D = \log_{10} O,$$

where D is the "density" and O the "opacity," the latter being defined as the ratio of the intensity of incident to transmitted light. This is measurable by the photometer, the actual measure being the deviation of the image of the recording thread, the movement of which has been calibrated by passing a beam through a standard test-plate. As to the requirements and construction of the test-plates, particulars are given in Mr. Baker's paper. The deviation shown by the thread when any portion of the plate is examined is thus read as D, or as any chosen function of D, say Δ, which may be found more convenient.

In the same way let T represent any suitable function of the exposure time of τ seconds.

Our interest is to derive the intensity of the source from observations of density and exposure time; let us then write

$$\text{(i)} \quad . \qquad \log (J/J_0) = a\Delta + b\mathrm{T} + \tfrac{1}{2}c\Delta^2 + d\Delta\mathrm{T} + \tfrac{1}{2}e\mathrm{T}^2 + \ldots,$$

where J is the required intensity and J_0 the intensity corresponding to $\Delta = 0$, $T = 0$. If there is any regular relation between intensity, density, and exposure time, an equation of the form (i) must subsist. It is supposed that sufficient terms of the development are taken, and the ranges of Δ, T are sufficiently restricted to secure a satisfactory convergence. The coefficients a, b, c, d, e, . . . are functions of the wave-length only, since the plate and its treatment in development, etc., are supposed unvaried. The equation thus stated contains no empiric postulate or debatable theory whatever; any observation or experiment may be interpreted as an evaluation of the coefficients.

By an algebraic reversion, which need not be written down, the same equation would give Δ, or the density, by means of a development proceeding in terms of T and $\log (J/J_0)$, and for a chosen series of values of Δ we should have the "curves of equal blackening."

The combined Hurter-Driffield and Schwarzschild theories may be put in the shape

$$\Delta = \gamma \log (J\tau^p/i),$$

where Δ is a function of the density, γ the "development constant," p Schwarzschild's index, and i the "inertia of the plate," all for a given wave-length. This relation may be read from equation (i), by omitting the terms in c, d, e, . . . and writing

$$\gamma = 1/a, \quad p = -b, \quad i = J_0 \tau_0^p, \quad T = \log(\tau/\tau_0),$$

if τ_0 is a standard exposure time. Hence any departures from this well-known theory may be looked for in values of c, d, e, . . . different from zero.

If the cross coefficients d . . . are zero, then by a change of variables to Δ_1, T_1, where

$$a\Delta_1 = a\Delta + \tfrac{1}{2}c\Delta^2 + \ . \ . \ .$$
$$bT_1 = bT + \tfrac{1}{2}eT^2 + \ . \ . \ .,$$

the general equation (i) may be reduced to the elementary form

(ii) . . . . $\log_{10}(J/J_0) = a\Delta_1 + bT_1$.

In this form Δ_1, T_1 would as a rule be functions of the wave-length, but for the densities here employed the substitution

$$\Delta_1 = \log_{10}(10^D - 1) = D + \log_{10}(1 - 10^{-D})$$

has been found by Mr. Baker to serve closely for all wave-lengths. Such a range may be held to lie between $-1 < \Delta < +1$, corresponding to $\cdot 04 < D < 1\cdot 04$. It will be taken as the definition of Δ hereafter. The substitution is empiric, and beyond a certain point it ceases to apply. But for initial densities, corresponding to $\Delta \to -\infty$, the equation (i) itself ceases to be applicable.

No similar substitution will be made in respect to T_1, but merely writing

$$T = \log_{10}(\tau^s/30^s),$$

the range of T employed in practice will lie between -1 and $+1$, corresponding to a range of exposure between 3 sec. and 300 sec.

Without anticipating any question of form, we may write

(iii) . . . $-p = \dfrac{\partial(\log J/J_0)}{\partial T} = b + d\Delta + eT$ . . . .

Mr. Baker has investigated the value of p, so defined, for the different makes and batches of plate employed separately for violet light ($\cdot 4\ \mu$) and for red light ($\cdot 6\ \mu$) (i) in terms of Δ, when $T = 0$, *i.e.* $\tau = 30$ sec.; (ii) in terms of T when $\Delta = 0$; and further, (iii) the continuous variation of p between these wave-lengths for $T = 0$, $\Delta = 0$.

From these investigations I conclude that from $\Delta = -1$ up to $\Delta = +0\cdot 7$ ($D = 0\cdot 8$) there is little evidence that p is other than constant for $T = 0$. For such a range, then, the cross coefficients d . . . are insensible. Beyond $\Delta = +1$ they certainly become sensible, and an increase in Δ, which measures the blackening, requires a more than proportionate increase in $\log J$, measuring the intensity of illumination. This is a well-known effect. The bearing upon the present work is that there

is a region over which the cross coefficients may be treated as zero, and the densities employed are confined to this region.

As regards any associated change of p with wave-length, this has been investigated for $T = 0$, $\Delta = 0$ over a range of λ. This is a study of the coefficient b as a function of λ. It appears from this that the rate of change of b with λ is very slight, and may be taken as negligible.

Further, as regards change in p for $\Delta = 0$, it appears that there is a clear diminution for increase in T; this diminution appears to be linear. In other words, the coefficient b being negative, e is positive; or, for a given blackening ($\Delta = 0$), the greater the value of T, the less is the associated value of log J.

The outcome of the investigation is that, in applying form (i), we have the coefficient $d = 0$; no coefficients beyond a, b, c, e are required; c may be absorbed into a by taking empirically

$$\Delta = D + \log_{10}(1 - 10^{-D}),$$

and b, e are of opposite signs and do not vary with λ.

The actual values of the coefficients will not be employed below, as the following process of reduction eliminates them.

Returning to equation (i) we have, for any wave-length,

$$\log J/J_0 = a\Delta + bT + \tfrac{1}{2}eT^2,$$

where a, b, e are functions of the wave-length. Hence, if two experiments are compared for the same wave-length,

$$\log J/J' = a(\Delta - \Delta') + b(T - T') + \tfrac{1}{2}e(T^2 - T'^2).$$

This equation permits an experimental calibration of the measures. Thus if experimental exposures are made for the same time and for ascertained ratios of intensity, the terms in b, e disappear, and an equation results,

$$\log J = a\Delta + M,$$

where M is an arbitrary constant. The corresponding values of J and Δ, plotted as a curve, give for the particular wave-length considered the interpretation of any observed density or the equivalent deviation of the thread as intensity *referred to an arbitrary unit.* This arbitrary unit is eliminated by the subsequent process of reduction.

Such curves can be constructed experimentally for a series of wave-lengths. They have been so constructed for each batch of plates employed for the wave-lengths ·6 μ and ·4 μ. The former appears by trial to work very well from ·540 μ onwards, and the latter to include ·486 μ. There is at present a gap between these points, extending over the green-yellow, but this has not raised any practical difficulty in the present research. Specimens are given in Plate 11 showing the alteration of scale when the screen is interposed.

Consider now the comparison of two stellar spectra. In order to compare the relative intensities of radiation for one star for any two chosen wave-lengths, with the same ratio for a second star, form the function

$$\log J_{\lambda_1} - \log J'_{\lambda_1} + \log J'_{\lambda_2} - \log J_{\lambda_2}.$$

This function has the property that any instrumental absorption for either wave-length affects J, J′ in the same measure, and is therefore eliminated. The same is true of any atmospheric absorption, so far as it may be considered unvaried from one exposure to another. As to this point, evidence of experience will be given later. Any fault in this respect is taken as an inevitable accidental error, and the function is adopted as a pure comparison of the relative intensities of radiation of the stars. I proceed to show that this function also eliminates the exposure times.

Considering the expression to which it must be equated, the exposure times for J, J′ are different, but of course they are the same for the two wave-lengths for each star. Hence we have

$$\log (J_{\lambda_1} J'_{\lambda_2} / J'_{\lambda_1} J_{\lambda_2}) = a_{\lambda_1}(\Delta_{\lambda_1} - \Delta'_{\lambda_1}) - a_{\lambda_2}(\Delta_{\lambda_2} - \Delta'_{\lambda_2}) + (b_{\lambda_1} - b_{\lambda_2})(T - T') + \tfrac{1}{2}(e_{\lambda_1} - e_{\lambda_2})(T^2 - T'^2).$$

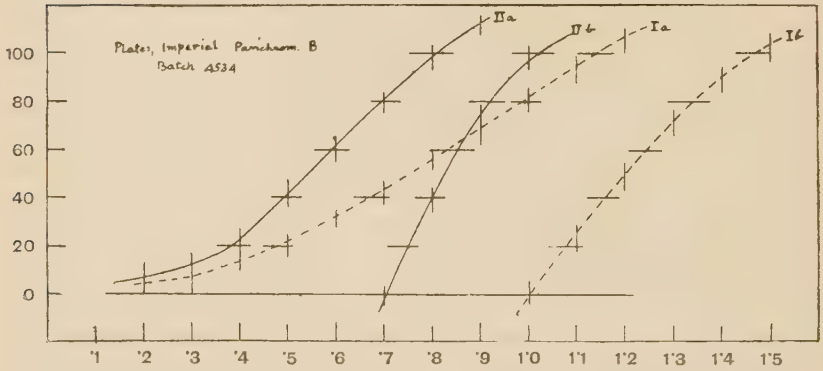


FIG. 1.—Interpretation Curve: *Ordinate*, Deviation; *Abscissa*, log J + constant.

·1*a* : wave-length ·4μ, unscreened.
1*b* : ,, ,, screened.
II*a* : ,, ·6μ, unscreened.
II*b* : ,, ,, screened.

If *b*, *e* do not vary with λ the exposure times are eliminated. In view of the large and inevitable atmospheric variations, any correction for the second line would be insensible, and the elimination is treated as complete. Thus the equation is read in the form

(iv) . . . $\log (J_\lambda / J'_\lambda) = a(\Delta_\lambda - \Delta'_\lambda) + N,$

where N is an arbitrary constant

$$= \log (I_\lambda / I'_\lambda) + N,$$

where I, I′ are derived from the adopted interpretation curves which have been determined experimentally to apply for the wave-length considered.

By differentiation we take

(v) . . $\frac{1}{J_\lambda}\frac{\partial J_\lambda}{\partial \lambda} - \frac{1}{J'_\lambda}\frac{\partial J'_\lambda}{\partial \lambda} = \frac{1}{I_\lambda}\frac{\partial I_\lambda}{\partial \lambda} - \frac{1}{I'_\lambda}\frac{\partial I'}{\partial \lambda} = \lambda^{-2} m,$ say.

This equation, which corresponds to (viii), (ix), pp. 184, 187, of the previous paper, is the basis for interpreting the photograms. The

actual procedure is the following. The ordinate of the photogram is measured for a series of wave-lengths, and the corresponding intensity as shown on the adopted interpretation curve is noted. The difference between this and the corresponding intensity for the same wave-length for the photograms of Polaris, taken on the same plate, is formed. That is to say, equation (iv) is formed, and N is then eliminated by charting the results for the range of wave-length and reading the gradient. The interpretation of the left-hand member so as to give information regarding the temperature of the star is hardly varied from the treatment given in the previous paper, but is briefly resumed below.

Physical Theory.

By the "effective temperature" of a star is understood that value of the temperature, T, which gives the best agreement between the measured radiation and Planck's formula for a black body. How nearly observed radiation may agree over its whole range of wave-length with such a formula must be a matter for examination. It is known in the case of the Sun that the agreement is not exact, the observed radiation being deficient in the ultra-violet and too strong in the infra-red. This applies to the entire disc, with which of course stellar determinations are concerned. But if, as seems most natural, the radiation from the centre of the disc is taken as standard, the values for the integrated disc will be modified by the progressive reddening of the zones as the margin is approached. This is explained, and a treatment given, in my previous paper. Thus if I_λ refers to the centre of the disc, we write

$$J_\lambda = \psi(\lambda T) I_\lambda = \psi(\lambda T) \,.\, \lambda^{-5} / \left(\exp \frac{c_2}{\lambda T} - 1 \right),$$

and ψ enters the equation as a computed correction. The basis for its computation is found in the known particulars for the Sun's disc. The resulting numbers are shown, *l.c.*, p. 201. It is evident from such considerations that some departure from black radiation must be expected in the case of a star, such a departure being a well-recognised feature for the case of the Sun. We must expect, then, in every case the presence of a qualifying function ψ, which may be called the feature associated with the stellar atmosphere, or the pressure feature, in the spectrum, whether we refer to the continuous spectrum or the line spectrum. It is a feature we should expect to change in the progression from a gaseous star to a condensed one. How closely the calculated numbers apply must be settled by trial, when sufficient evidence is accumulated. But whatever the proper value for ψ, the equation (v) leads to

$$\text{(vi)} \quad . \quad . \quad . \quad \left(\frac{c_2}{T} N + \chi_T \right) - \left(\frac{c_2}{T'} N' + \chi_{T'} \right) = m,$$

where

$$N = 1 / \left(1 - \exp \frac{-c_2}{\lambda T} \right), \quad \chi = -d(\log \psi)/d(1/\lambda),$$

and

$$m = -\frac{d \log (I/I')}{d(1/\lambda)}.$$

The function on the left changes only slightly with the wave-length considered. If, then, log (I/I′), as read by comparison of the measures of the two stars from the adopted interpretation curve, is plotted with $1/\lambda$ as abscissa, the gradient m at all points should be nearly the same. By this means the whole sequence of measures may be taken into account visually and together, and a single mean gradient adopted, which is taken as referring to wave-length ·5 μ. At the same time an estimate may be formed whether the departures from uniformity have any systematic meaning or only an accidental one. Some remarks will be made later on the results, but it may be said at once that experience shows the method to be a satisfactory one.

The following table and figure show the related values of T, c_2/T, and $\frac{c_2}{T}N + \chi_T$, $\equiv \Phi$, say.

The value adopted for c_2 is 14600.

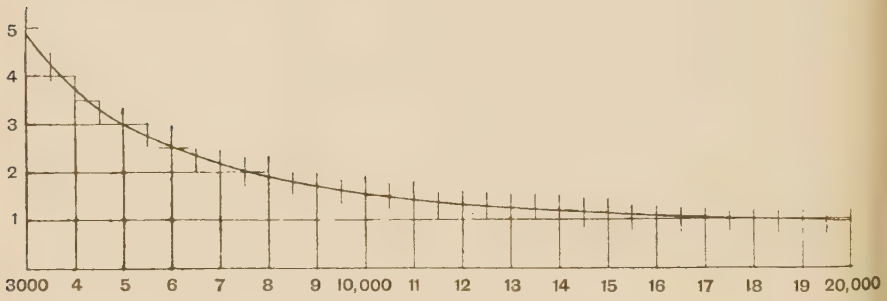


FIG. 2.—*Abscissa*, Temperature; *Ordinate*, Gradient.

TABLE I.

T.	c_2/T.	Φ.	T.	c_2/T.	Φ.	T.	c_2/T.	Φ.
3000°	4·87	4·88	9,000°	1·62	1·69	15,000°	·98	1·12
3500	4·17	4·21	9,500	1·54	1·61	15,500	·94	1·10
4000	3·65	3·71	10,000	1·46	1·54	16,000	·91	1·08
4500	3·24	3·29	10,500	1·39	1·47	16,500	·88	1·06
5000	2·92	2·99	11,000	1·33	1·41	17,000	·85	1·04
5500	2·65	2·74	11,500	1·27	1·36	17,500	·83	1·02
6000	2·43	2·52	12,000	1·22	1·32	18,000	·81	1·01
6500	2·25	2·33	12,500	1·17	1·28	18,500	·79	·99
7000	2·09	2·16	13,000	1·12	1·24	19,000	·77	·98
7500	1·95	2·02	13,500	1·08	1·20	19,500	·75	·96
8000	1·83	1·89	14,000	1·04	1·17	20,000	·73	·95
8500	1·72	1·78	14,500	1·01	1·14	30,000	·48	·77
						∞	0	·5

The correction χ for the reddening of the margin of the disc operates in the direction of extending the range of Φ for a given temperature range, especially for the lower temperatures. This would correspond to a progressive blotting out of the violet and blue light as the temperature fell.

It will be seen that this curve becomes increasingly weak for the determination of T, for an observed value of Φ, as T rises. Thus at 4000° an error +·05 produces a correction of −500°, and −·05 produces +600°; but at 16,000° the corrections for the same errors would be respectively −1250° and +1250°. This arises from a double cause. First, T is found from its reciprocal, c_2/T; and, of more physical interest, to specify c_2/T from the summit of the radiation curve, which is the essence of the physical observation, becomes increasingly difficult when the summit is crowded towards the shortest wave-lengths. Values marked as 20,000° or upwards must be regarded merely as tokens. Fortunately, another method is here available, in the appearance of lines belonging to ionised elements.

It is a necessary logical step to adopt, at any rate for trial, the temperature of one star. It will be seen below that the test of this single trial or adoption is shared among the whole mass of observations, so that there hardly remains any arbitrary element in it. Adopting as before the temperature of Capella at 5500°, and entering this table with the observed difference in Φ for Capella relative to Polaris, of ·48 (*cf.* Table III., p. 225, General List of Stars), we find the temperature of Polaris to be 6700°; or, finally, if a zenith distance correction of −·08 is applied to the observed gradient, as determined on p. 235, the value becomes 6500°. The value assigned in my previous paper was 6800°. The other stars are read relative to Polaris, and their temperatures are determined in agreement with it.

Material Employed.

I proceed to describe the actual material. To obtain the most consistent results it is desirable to vary the plates employed, and the method of treating them, as little as possible. At this stage, however, with the method on its trial, it was thought advisable to sacrifice a little consistency, in order to gain experience. Accordingly, several batches of plates of three different makes were used, *Ilford Spectrum Panchromatic*, *Wellington Spectrum Panchromatic*, and *Imperial Panchromatic B*. These were in some cases used as delivered; in other cases they were washed in tap water for different periods and dried again, before exposure; in others again they were washed after exposure. The latter produces little or no effect, but washing before exposure greatly increases the red sensitivity, and in less degree the general sensitivity also, for equal development time. The plates may be kept without deterioration for at least a fortnight after washing and drying. The method presents no serious troubles if a proper drying chamber, through which a blast of air is driven, is available.

It will be noticed also that the same developer has not been adhered to throughout, nor the same temperature or time of development.

TABLE II.

List of Plates.

No.	Description.	Date of Exposure.	Exposure Note.	Development Note.
80	Ilford 5844 E	1923 Jan. 22	Sky very changeable at first, afterwards kept pretty constant until end of No. 82.	Not washed. Developed at 57° with No. 82.
81	,, ,,	,,		,, ,, 60°.
82	,, ,,	,,		,, ,, 57°.
83		Feb. 8		,, ,, 59° with No. 84.
84		,,		Washed. ,, 59°.
85		,,		,, ,, 59°.
86		Feb. 23	Clouded after 2 exposures.	,, ,, 58°.
87		Mar. 5		,, Feb. 9. Developed at 60°.
88	Ilford 5858 E	,,		,, ,, ,, 60°.
89	,, 5844 E	Mar. 6	Thin cloud came up.	,, ,, ,, 59°.
90	,, 5858 E	Mar. 20	Clouded after 3 exposures.	,, ,, ,, 57°.
92	,, ,,	Mar. 24	Sky changeable: apparently a mist up to about 40°.	,, Mar. 24. ,, 58°.
93	,, ,,	Mar. 29	Bright moonlight, but sky good: light easterly wind.	
94	,, ,,	,,	,, ,,	,, ,,
95	,, ,,	,,	,, ,,	,, ,,

96	Ilford 5858 E	Apr. 5	Clouded after 3 exposures.	Washed.	Mar. 26.	Developed at	57°.	
97	,, ,,	Apr. 17		,,	,,	,,	57°.	
98	,, ,,	Apr. 23		,,	,,	,,	60°.	
99	,, 5869 E	Sept. 7		,,	Sept. 1	,,	60°.	
100	,, 5891 E	Sept. 15	Windy but clear.	,,	Sept. 14	,,	58°.	
101	,, ,,	Sept. 18	Wind gusty—west.	,,	,,	,,	56°.	
103	,, ,,	Oct. 3		,,	in running water after exposure, $1\frac{1}{2}^{h}$.		Developed at	55°.
104	,, ,,	Oct. 5	Cloudy.	,,	in distilled water	,, ,,	30^{m}. ,,	60°.
105	,, ,,	Oct. 11	Thin cloud in east, alt. 45°.	,,	10^{m} after exposure.			
106	,, 5895 E	,,		,,	,,	,,		
107	,, 5891 E	Oct. 24		,,	20^{m}	,,	,,	
108	,, ,,	Oct. 26	Thin cloud to south.	,,	40^{m}	,,	,,	
109	,, ,,	Oct. 30	Cloudy.	Not washed.				
110	,, ,,	Oct. 31	,,	,,				
111	,, 5895 E	Nov. 4	Windy : definition fair.	,,	Developed at 55°.			
112	,, 5891 E	Nov. 6		,,	,,	56° with No. 113.		
113	,, ,,	,,		,,				
115	,, 5895 E	Nov. 8	Some mist.	,,	,,	55° with No. 116.		
116	,, ,,	,,	Sky misty in places.	,,				
117	Well. Spec. Pan.	Nov. 14	Clouded and snowed.		,,	60°, 5^{m} in MQ.		
118	,, ,,	,,		,,	,,	57°, $6\frac{1}{2}^{m}$.		

TABLE II.—*continued.*

No.	Description.	Date of Exposure.	Exposure Note.	Development Note.
119	Well. Spec. Pan.	Nov. 14		Not washed. Developed at 57°, $6\frac{1}{2}^{m}$.
120	,, ,,	Nov. 23	Bright moon : clouded.	,, ,, 66°, 5^{m}.
121	,, ,,	Nov. 24	Bright moon.	Washed before exposure, 2^{h}. Developed at 66°.
122	,, ,,	,,	,, ,,	,, ,, ,, ,, ,,
123	Imp. Pan. B	Nov. 29	Sky slightly hazy : frosty.	,, after exposure, $1\frac{3}{4}^{h}$. Developed at 64°, $3\frac{1}{2}^{m}$ in MQ.
124	,, ,,	,,	,, ,, ,,	,, before ,, $1\frac{3}{4}^{h}$. ,, 64°, $3\frac{1}{2}^{m}$ in MQ, with 123.
125	,, ,,	Dec. 3	Sky misty at times.	,, Nov. 29. Developed at 64° in MQ, 4^{m}.
126	Well. Spec. Pan.	,,	Probably thin cloud.	,, ,, ,, 64° ,, 4^{m}.
127	Imp. Pan. B	Dec. 6	Clouded.	,, Dec. 3. ,, 64° ,, 4^{m}.
128	Well. Spec. Pan.	Dec. 8		,, ,, ,, 64° ,, 4^{m}.
129	Imp. Pan. B. Batch 4534	,,	Inclined to be foggy.	,, ,, ,, 64° ,, 4^{m}.
130	,, ,,	Dec. 10, 13	In dark slide from Dec. 10 to 13.	,, Dec. 10, 2^{h}. ,, 64° ,, $3\frac{1}{2}^{m}$.
131	,, ,,	Dec. 14		,, ,, ,, 64° ,, $3\frac{1}{2}^{m}$.
132	,, ,,	Dec. 18	Clouds came up.	,, ,,
133	,, ,,	1924 Mar. 4		,, Jan. 19.
134	,, ,,	Mar. 29	Clouded.	,,
135	,, ,,	Apr. 14	Bright moon.	,,
136	,, ,,	Apr. 15	Bright moon : sky good.	,,
139	,, ,,	Sept. 24	Thin cloud up to 30° : variable.	,, Sept. 19, $\frac{3}{4}^{h}$. ,, 65° in pyro, 4^{m}.

141	Imp. Pan. B. Batch 4534	Sept. 27		Washed Sept. 27.		Developed at 65° in pyro, $3\frac{1}{2}^{m}$.	
143	,, ,,	Oct. 15	Mist drifting, heavy dew, moon.	,,	Oct. 10, $1\frac{3}{4}^{h}$.	Developed in MQ.	
144	,, ,,	Oct. 16	Sky clear; bright moon in east.	,,	,,	,,	,,
145	,, ,,	Oct. 23	Apparently clear, but definition poor.	,,	,,	,,	at 65° in MQ for $2\frac{1}{8}^{m}$.
148	,, ,,	Oct. 24	Clouded.	,,	Oct. 24, 1^{h}.	Developed in MQ.	
150	,, ,,	Nov. 3	Sky variable; clouded.	,,	Oct. 31.	,,	,,
151	,, ,,	Nov. 10	Sky misty; drifting cloud; moon.	,,	,,	,,	,,
152	,, ,,	,,	Clouded.	,,	Nov. 10.	,,	,,
153	,, ,,	Nov. 13	Sky clear.	,,	,,	,,	,,
154	,, ,,	,,	Thin cloud stopped work.	,,	,,	,,	,,
155	,, 4585	Nov, 24	Sky good; wind; drifting cloud.	,,	Nov. 19.	,,	,,
156	,, ,,	Nov. 28	Thin cloud formed.	,,	,,	,,	,,
157	,, ,,	Dec. 14	Moon in east; sky apparently good.	,,	Dec. 13.	,,	,,

These experiments may be considered to have led to a pretty well-established standard method for future use.

No variation has been made in the method of taking the plates, except that higher densities are allowed, as already explained. If the night allowed, six spectra were taken on each plate, the first and last being that of Polaris. The plates were taken by Mr. Storey.

From these plates the photograms were prepared by Mr. Baker with the Koch microphotometer. The plan was the same as heretofore, except that with the greater densities employed, when the trace ran up towards a summit where it would become featureless, the scale was broken by inserting a screen of ascertained density in the path of the light which passes to the balancing cell. To match this broken scale, the interpretation curve is duplicated, as appears in fig. 1.

The measurement of the photograms was made by myself.

General List.

The following table contains the outcome of the individual observations, together with related particulars, as follows :—

Column 1 : serial number. Column 2 : number in Boss P.G.C. Column 3 : name of star and absolute magnitude ; where the latter has no reference attached, it is derived from Adams ; in the other cases (*Ed.*) refers to Edwards, *M.N.*, 84, 366 (1924), and (*Ab.*) to Abetti, *Oss. Arcetri*, fasc. iv., 41 (1924). Column 4 : spectrum first as given in the Henry Draper Catalogue, and then on the Mount Wilson system ; for the latter, from types F onwards there are two determinations, the first estimated, the second measured. Column 5 : the number of the plate and the number of the spectrum on the plate. Column 6 : the zenith distance of the star when observed. Column 7 : the observed gradient taken as explained above, for wave-length ·5 μ, relative to Polaris. Column 8 : the mean of the observed gradients, and the concluded temperature of the star ; the latter is derived from Table I., fig. 2, but an allowance, which is explained later, is applied on account of the zenith distance. For the cases where the temperature is enclosed in [ ], refer to the Notes. Column 9 : the corresponding value of c_2/T for the star, minus the same quantity for Polaris, as given by Hertzsprung, *Ann. Sterrewacht te Leiden*, 14, 1 (1922), "Mean Colour Equivalents of 734 Stars" ; further reference will be made to this column below. Column 10 : the same, as given by J. Wilsing, "Effektive Temperaturen von 199 Sternen," *Potsdam Astrophys. Obs.*, No. 74 (1919).

TABLE III.

General List of Stars.

1	2	3	4	5	6	7	8	9	10
Number. Ital.	Number. Boss.	Name. Abs. Mag.	Spectrum. H.D. : M.W.	Plate.	Zen. Distance.	Observed Gradient.	Mean Gradient. Temperature.	*Cf.* Hertzsprung.	*Cf.* Wilsing.
1	10	*α* And.	A*op* : A1*n*	100/4	46°	*1·210*	*1·057*	*1·07*	*1·05*
		1M·1		144/3	33	*1·100*			
				152/2	35	*·860*	12,600°		
2	12	*β* Cas.	F5 : F2, F4	100/3	32	*·560*	*·493*	*·35*	*·20*
		1M·6		144/4	8	*·560*			
				152/3	14	*·360*	7,800°		
3	27	*γ* Peg.	B2 :	104/4	45	*1·556*	*1·433*	*1·33*	*1·21*
		2M·9 (Ed.)		113/4	43	*1·310*	25,000°		
4	199	*γ* Cas.	B*op* :	100/5	32	*1·975*	*1·827*	*1·07*	*·44*
		..		113/3	13	*1·390*	[30,000°]		
				116/4	18	*2·115*			
				120/3	11				
5	259	*β* And.	Mo : Mo, G8	101/4	39	1·812	2·102	1·74	1·94
		0M·5		110/3	26	2·212			
				117/5	34	2·056	3,300°		
				122/6	33	2·328			
6	314	*δ* Cas.	A5 : A3*n*	101/3	29	*·889*	*·977*	*·65*	*·08*
		2M·0		124/5	7	*1·064*	10,700°		
7	325	*α* UMi.	F8 : F9, F8	all	34	..	..	..	..
		3M·0					6,500°		
8	419	*ε* Cas.	B3 : B5*n*	128/4	14	*1·540*	*1·406*	*1·09*	
		0M·5		131/4	16	*1·272*	17,400°		
9	428	*β* Ari.	A5 :	101/5	55	*·736*	*·734*	*·62*	*·75*
		..		113/5	44	*·732*	10,300°		
10	468	*γ*1 And.	K0 : K2, G9	104/5	26	1·280	1·270	1·51	1·51
		0M·1		106/3	26	1·260	4,000°		
11	477	*α* Ari.	K2 : K2, K0	111/3	36	1·224	1·128	1·09	1·10
		0M·7		118/5	35	1·032	4,300°		

TABLE III.—*continued.*

1	2	3	4	5	6	7	8	9	10
Number. Serial.	Boss.	Name. Abs. Mag.	Spectrum. H.D. : M.W.	Plate.	Zen. Distance.	Observed Gradient.	Mean Gradient. Temperature.	*Cf.* Hertzsprung.	*Cf.* Wilsing
					°				
12	482	β Tri.	A5 :	126/3	23	*1·060*	*·948*	*·65*	*·3*
		$1^{M}\cdot2$ (Ab.)		128/5	23	*·920*			
				131/4	25	*·864*	10,700°		
13	691	α Cet.	Mo : Mo, G9	111/4	57	2·540	2·345	1·77	1·72
		$0^{M}\cdot1$		119/3	54	2·150	3,200°		
14	694	γ Per.	F5*p* : F9, F9	129/3	11	·244	·196	·24	·48
		$0^{M}\cdot1$		131/5	8	·148	5,700°		
15	708	β Per.	B8 :	110/4	28	*·940*	*1·144*	*·88*	*1·2*
		$0^{M}\cdot4$ (Ed.)		111/5	18	*1·348*	12,800°		
16	869	η Tau.	B5*p* : B8*n*	129/5	32	*·948*	*·923*	*1·04*	*1·0*
		$0^{M}\cdot3$		150/5	50	*·732*			
				157/2	34	*1·088*	11,300°		
17	910	ε Per.	B1 : B1*n*	129/4	21	*1·228*	*1·228*	*1·12*	*·6*
		$2^{M}\cdot0$					14,000°		
18	1077	α Tau.	K5 : K4, K2	119/4	45	1·952	1·931	1·89	1·57
		$1^{M}\cdot0$		157/3	43	1·910	3,400°		
19	1246	α Aur.	G0 : G1, G1	94/2	47	·508	·481	·18	*·5*
		$0^{M}\cdot1$		94/3	43	·560			
				103/6	44	·388	5,500°		
				115/5	43	·372			
				118/3	43	·692			
				123/1	43	·308			
				133/3	14	·208			
				141/4	62	·732			
				148/4	60	·586			
				148/5	47	·456			
20	1250	β Ori.	B8*p* :	83/3	67	*1·024*	*·936*		
		..		83/4	67	*·888*			
				83/5	67	*1·040*	14,800°		
				84/3	67	*·848*			
				84/4	67	*·948*			
				84/5	68	*1·108*			
				133/4	69	*·708*			

TABLE III.—*continued.*

1	2	3	4	5	6	7	8	9	10
Number. Serial.	Boss.	Name. Abs. Mag.	Spectrum. H.D. : M.W.	Plate.	Zen. Distance.	Observed Gradient.	Mean Gradient. Temperature.	*Cf.* Hertzsprung.	*Cf.* Wilsing.
21	1303	γ Ori.	B2 : B2*s*	88/3	57°	*1·368*	*1·368*	*1·26*	*1·64*
		$2^{M}\cdot0$					25,000°		
22	1304	β Tau.	B8 : B9*s*	93/4	44	*1·588*	*1·688*	*1·12*	*1·13*
		$0^{M}\cdot2$		93/5	44	*1·788*	[30,000°]		
23	1339	δ Ori.	Bo : B1*n*	90/2	70	*1·420*	*1·280*	*1·34*	
		$2^{M}\cdot4$		157/4	63	*1·140*	25,000°		
24	1370	ε Ori.	Bo : B1*n*	93/2	72	*1·512*	*1·223*	*1·24*	
		$2^{M}\cdot4$		93/3	72	*1·268*	25,000°		
				157/5	62	*·890*			
25	1398	ζ Ori.	Bo : Bo*n*	90/3	72	*1·084*	*1·084*	*1·36*	
		$2^{M}\cdot8$					20,000°		
26	1468	α Ori.	Mo : Mo, G3	88/4	61	2·125	2·138	2·02	2·19
		$3^{M}\cdot8$		88/5	61	2·130	3,400°		
27	1478	β Aur.	Ao*p* : Ao*np*	126/4	13	*1·024*	*1·024*	*·93*	*·39*
		$1^{M}\cdot5$					11,100°		
28	1732	α CMa.	Ao : A2*s*	85/1*a*	78	*·604*	*·746*	..	..
		$0^{M}\cdot9$		85/1*b*	78	*·940*			
				87/3	75	*·868*	12,800°	..	..
				87/4	74	*·608*			
				87/5	74	*·712*			
29	2008	α CMi.	F5 : F3, F5	85/3	52	*·320*	*·390*	*·32*	*·58*
		$3^{M}\cdot2$		85/4	52	*·460*	8,300°		
30	2031	β Gem.	Ko : G9, G8	81/4	35	1·060	1·154	·83	·40
		$1^{M}\cdot7$		81/5	35	1·120			
				82/2	33	1·152	4,200°	..	..
				82/3	32	1·168			
				82/4	31	1·268			
31	2404	ι UMa.	A5 : A4*n*	134/3	8	*·948*	*·928*	*·53*	*·83*
		$2^{M}\cdot2$		136/3	16	*·908*	10,200°		
32	2552	θ UMa.	F8 : F5, F6	135/3	9	*·040*	*·200*	*·13*	..
		$3^{M}\cdot0$		136/4	20	*·360*	7,100°		..

TABLE III.—*continued.*

1	2	3	4	5	6	7	8	9	10
Number. Serial.	Boss.	Name. Abs. Mag.	Spectrum. H.D. : M.W.	Plate.	Zen. Distance.	Observed Gradient.	Mean Gradient. Temperature.	*Cf.* Hertzsprung.	*Cf.* Wilsing.
33	2930	β UMa. $1^{M}\cdot0$	Ao : A3*s*	96/2	30°	*1·490*	*1·448*	*1·00*	·9
				97/3	5	*1·405*	20,000°		
34	2933	α Uma. $0^{M}\cdot4$	Ko : G8, G8	89/4	17	1·030	·894	·87	·36
				96/3	6	·580	4,500°		
				148/3	61	1·072			
35	3117	γ UMa. $0^{M}\cdot9$	Ao : A*on*	95/5	5	*1·208*	*1·356*	*1·04*	*1·2*
				97/2	5	*1·504*	16,200°		
36	3363	ε UMa. $0^{M}\cdot9$	A*op* : A2*s*	89/2	34	*1·225*	*1·318*	*·80*	*1·4*
				89/3	35	*1·290*			
				95/4	7	*1·440*	16,400°		
37	3370	12 CVn. $0^{M}\cdot6$	A*op* : A1*s*	98/3	20	*1·360*	*1·360*	*·83*	*·9*
							16,800°		
38	3474	ζ^{1} UMa. $1^{M}\cdot1$	A*op* : A2*s*	95/3	14	*1·340*	*1·353*	*·91*	*1·1*
				97/4	12	*1·365*	16,400°		
39	3566	η UMa. $0^{M}\cdot8$ (Ed.)	B3 :	95/2	22	*1·530*	*1·615*	*1·24*	*1·22*
				97/5	12	*1·700*	25,000°		
40	3596	η Boo. $2^{M}\cdot8$	Go : F9, Go	98/5	40	·192	·192	·05	·07
							6,000°		
41	3662	α Boo. $1^{M}\cdot2$	Ko : Ko, G9	92/3	51	1·224	1·341	1·06	1·32
				92/4	50	1·212			
				92/5	48	1·565	4,200°		
				94/4	55	1·195			
				94/5	54	1·510			
42	3809	β UMi. $0^{M}\cdot5$	K5 : K4, Ko	98/2	29	1·860	1·880	1·64	1·31
				116/3	19	1·965			
				122/3	20	1·815	3,400°		
43	3961	α CrB. $0^{M}\cdot9$	Ao : A*on*	98/4	56	*1·135*	*1·135*	*1·09*	*1·37*
							16,900°		
44	4192	η Dra. $0^{M}\cdot7$	G5 : G6, G5	107/3	32	·536	·627	·66	·45
				108/3	42	·718	5,100°		

TABLE III.—*continued.*

1	2	3	4	5	6	7	8	9	10
Number. Serial.	Boss.	Name. Abs. Mag.	Spectrum. H.D. : M.W.	Plate.	Zen. Distance.	Observed Gradient.	Mean Gradient. Temperature.	*Cf.* Hertzsprung.	*Cf.* Wilsing.
45	4443	β Dra.	Go, G*op*, F8	105/4	39°	·488	·676	·58	·39
		$3^{M}\cdot5$		143/3	38	·840			
				145/3	45	·700	5,000°		
46	4459	α Oph.	A5 : A*on*	105/3	67	*·724*	*·656*	*·68*	*·79*
		$0^{M}\cdot7$		145/2	67	*·588*	10,900°		
47	4541	γ Dra.	K5 : K5, K1	99/3	20	2·185	1·973	1·84	1·54
		$0^{M}\cdot6$		112/4	32	1·760	3,400°		
48	4722	α Lyr.	A0 : A1*s*	118/4	58	*·892*	*·938*	*1·00*	*1·04*
		$0^{M}\cdot6$		123/3	45	*1·052*			
				139/2	35	*·828*	11,600°		
				139/3	38	*1·000*			
				139/4	42	*·944*			
				139/5	45	*·912*			
49	4858	ζ Aql.	A0 : B9*n*	103/3	57	*1·152*	*1·224*	*·87*	*1·36*
		$0^{M}\cdot8$		115/3	64	*1·295*	20,000°		
50	5047	γ Aql.	K2 : K3, G9	105/5	52	1·725	1·650	1·64	1·57
		$0^{M}\cdot3$		112/3	52	1·575	3,800°		
51	5048	δ Cyg.	A0 : A1*n*	103/4	32	*1·344*	*1·348*	*·85*	*·85*
		$1^{M}\cdot1$		112/5	26	*1·352*	17,400°		
52	5229	γ Cyg.	F8*p* : G*op*, G0	99/5	17	·000	·044	·28	*·29*
		$3^{M}\cdot0$		117/3	23	*·048*			
				153/3	23	·116	6,200°		
				156/2	33	·108			
53	5320	α Cyg.	A2*p* : A2*s*	99/4	12	*1·248*	*·989*	*·85*	*1·05*
		..		117/4	22	*1·590*			
				123/4	27	*1·280*	10,900°		
				153/5	24	*·502*			
54	5336	ε Cyg.	K0 : G7, G7	150/2	27	·996	·823	·81	·71
		$0^{M}\cdot8$		151/2	29	·680			
				153/4	30	·792	4,600°		
55	5480	α Cep.	A5 : A2*n*	150/3	9	*·680*	*·730*	*·69*	*·47*
		$1^{M}\cdot3$		151/3	13	*·780*	8,900°		

TABLE III.—*continued.*

1	2	3	4	5	6	7	8	9	10
Number. Serial.	Boss.	Name. Abs. Mag.	Spectrum. H.D. : M.W.	Plate.	Zen. Distance.	Observed Gradient.	Mean Gradient. Temperature.	*Cf.* Hertzsprung.	*Cf.* Wilsing.
					°				
56	5532	β Cep.	B1 : B1*s*	121/4	25	*1·368*	*1·308*	*1·28*	*1·52*
		$2^{M}\cdot3$		124/3	26	*1·248*	16,500°		
57	5584	ε Peg.	K0 : K0, G4	153/2	48	1·675	1·818	1·52	·67
		$0^{M}\cdot3$		155/2	49	1·960	3,600°		
58	5714	ζ Cep.	K0 : K2, G4	124/4	24	1·730	1·710	1·39	..
		$0^{M}\cdot5$		125/4	22	1·690	3,600°		
59	5807	δ Cep.	G0 : G0, F9	108/4	1	·000	·067	·34	..
		$2^{M}\cdot7$		109/3	8	·016			
				123/5	8	·280	6,100°		
				125/3	10	·238			
				127/3	8	·000			
				128/3	8	·060			
				130/5	13	·252			
				131/2	27	·335			
				132/2	10	·045			
				150/4	5	*·008*			
				151/5	1	·184			
				154/2	12	*·392*			
				155/5	18	*·144*			
60	5891	ι Cep.	K0 : K0, G9	132/3	19	·932	·892	·71	..
		$1^{M}\cdot1$		143/4	15	·852	4,500°		
61	5933	o And.	B3 : B7*n*	132/4	27	*1·052*	*1·152*	*1·06*	*1·00*
		$0^{M}\cdot0$		143/4	18	*1·252*	12,800°		
62	5940	β Peg.	M0 : M2, G7	122/4	50	2·300	2·185	1·94	2·59
		$0^{M}\cdot0$		144/2	31	2·235			
				151/4	30	1·790	3,200°		
				155/4	33	2·415			
63	5944	α Peg.	A0 :	154/3	43	*1·128*	*1·114*	*1·10*	*1·21*
		$0^{M}\cdot1$ (Ab.)		155/3	43	*1·100*	14,200°		
64	6078	γ Cep.	K0 : K1, K1	115/4	21	·988	·926	·95	1·09
		$2^{M}\cdot3$		121/3	21	·864	4,500°		

Remarks and Notes.

I would first remark upon the intercomparison of the observations of Polaris. In order to examine the character of the dispersion that might be expected to occur in the measures without assignable cause, where there were two observations of Polaris on the same plate, the second was referred to the first, and the relative gradient ascertained, as though it had been a different star.

The following results were found :—

TABLE IV.

Dispersion of Measures of Polaris.

1	2	1	2	1	2
No. of Plate.	Gradient. Last *minus* First.	No. of Plate.	Gradient. Last *minus* First.	No. of Plate.	Gradient. Last *minus* First.
80/5, 3	·084	105/6, 2	·060	132/5, 1	·088
80/6, 5	·176	110/5, 2	·121	132/6, 5	·076
83/6, 2	·052	111/6, 2	·100	135/4, 2	·168
85/6, 2	·072	112·6, 2	·278	136/5, 2	·080
86/2, 1	·128	113/	·100	136/6, 5	·112
87/6, 2	·072	116/5, 2	·048	139/2, 1	·084
88/6, 1	·000	118/6, 2	·000	139/6, 2	·024
92/6, 1	·296	121/5, 2	·276	143/6, 1	·148
93/6, 1	·572	122/5, 2	·200	144/5, 1	·008
94/6, 1	·120	123/6, 2	·046	145/4, 1	·168
95/6, 1	·120	124/6, 2	·196	150/6, 1	·280
97/6, 1	·008	125/5, 2	·180	151/6, 1	·084
98/6, 1	·040	128/6, 2	·120	153/6, 1	·160
99/6, 2	·212	129/6, 2	·008	155/6, 1	·000
101/6, 2	·044	130/6, 2	·014	157/6, 1	·000
103/6, 2	·004	131/6, 1	·100		

It will be remarked at once that they are predominantly negative. Thirty-three are negative, compared to ten positive and four zero. This could be interpreted by saying that as a rule the night becomes more transparent to the blue in comparison to the red during the period, 1 to 2 hours, while the plate was being taken. This must be taken as a dependable effect, and has been treated in the comparisons of other stars with Polaris by using values for the latter interpolated between its first and last photograms.

The mean value, regardless of sign, is ·115. Five cases out of forty-seven exceed twice this amount, and one of these, Plate No. 93, is nearly five times as much. In this case the anomaly is very evident to the eye, and the plate could properly have been rejected. It is, however, carried into the discussion in order to illustrate to what degree such an instance distorts the results.

The different comparisons of any other star with Polaris may also be examined for dispersion. If there are n such comparisons, I take the whole dispersion, divided by $(n - 1)$, as the measure, and find an average of ·13. It appears, therefore, that the comparison of a star with the photogram of Polaris is little less reliable than a com-

parison with a repeated photogram of itself. The differences that are present might be due to patchiness in the plate, but they are believed to be caused chiefly by transient changes in the transparency of the sky. In any case it would seem that they could be eliminated by repetition, and this is confirmed by the close agreement between the measures of the present paper with the results given in my former one.

I next give a chart showing the mean gradient of each star, column 8 of Table III., plotted against the Mount Wilson spectral class, the estimated class being used from F onwards. Where no M.W. class was available, H.D. was employed.

The serial number of each star is given on one side. The continuous line represents an estimated standard place. The chief point of general interest in this chart is to ascertain how far the dispersion of the dots from standard place represent a real difference between the temperature classification and the spectral classification.

First, it may be conceded that the line representing standard place cannot be much displaced. It covers a range of gradient relative to Polaris from $-1{\cdot}40$ to $+2{\cdot}40$ approximately. Referring to fig. 2, attached to Table I., p. 218, it will be seen that if the upper temperature be taken at about 20,000°, these numbers place Polaris close to the position 6500°, which agrees with an adopted value of 5500° for Capella, when the observed gradient ·48 for the latter is modified by the Z.D. correction $-{\cdot}08$ as given on p. 235.

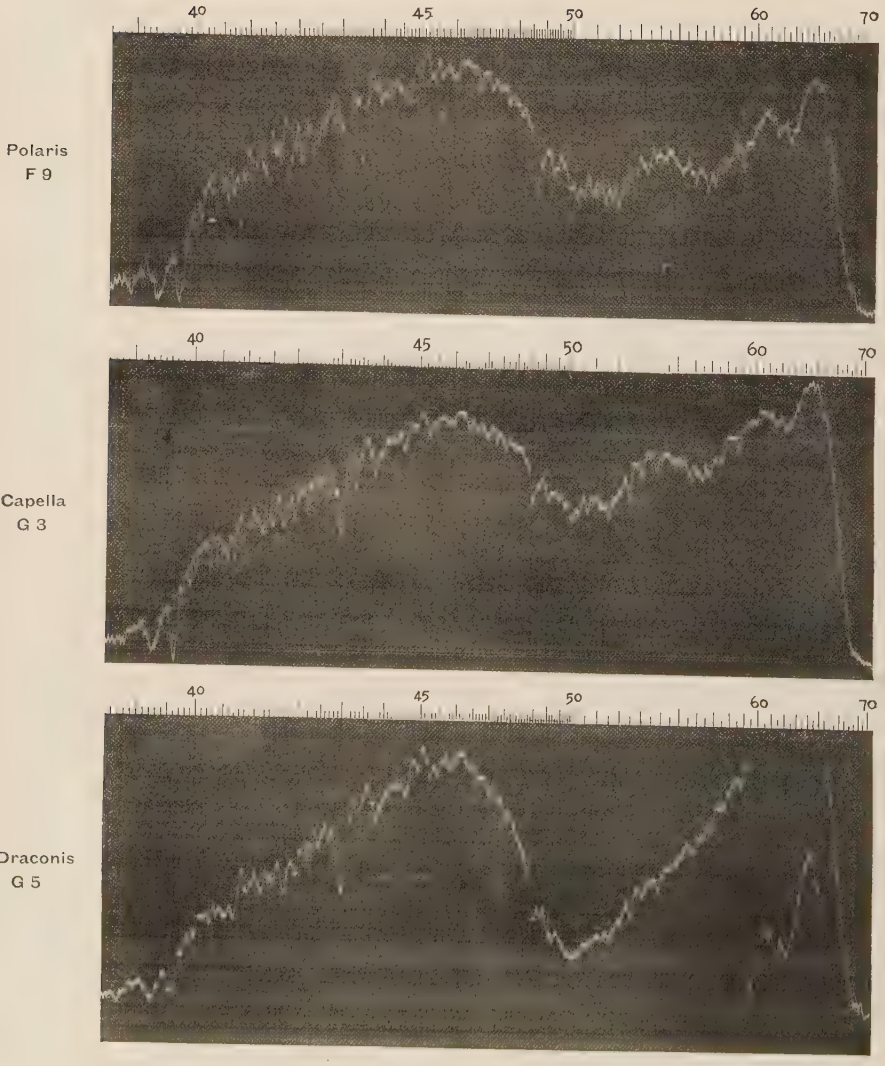
The same position would ascribe to the Sun, of class Go, a temperature of 6200°, which is precisely what the position of the summit of its radiation curve assigns.*

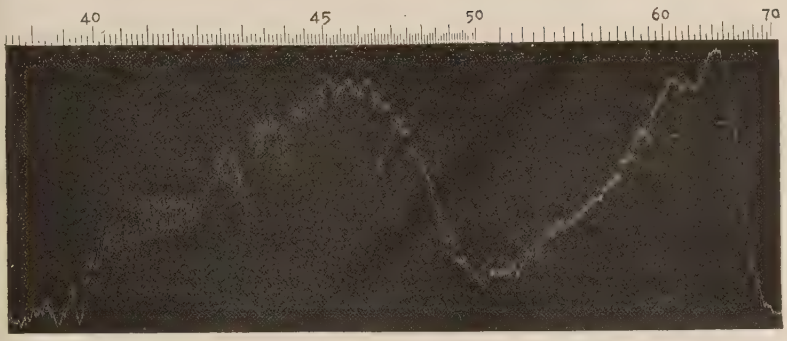
Consider then this standard place as fairly certain, and examine the departures from it. First, it is perhaps not recognised that even for these prominent stars there exists no agreed and reliable spectral classification. The attached figures show H.D. plotted against M.W. (estimated), and M.W. (estimated) plotted against M.W. (measured). It is evident that one cannot agree with them all. Each of these classifications is made on the basis of the intensity of certain lines. I am not at present engaged in considering measures of that kind upon the photograms, but for illustration I attach reproductions of the photograms of the following six cases, showing the spectral class which would be suggested for these stars, if the ascertained gradient is accepted and they are placed upon the standard line.

TABLE V.

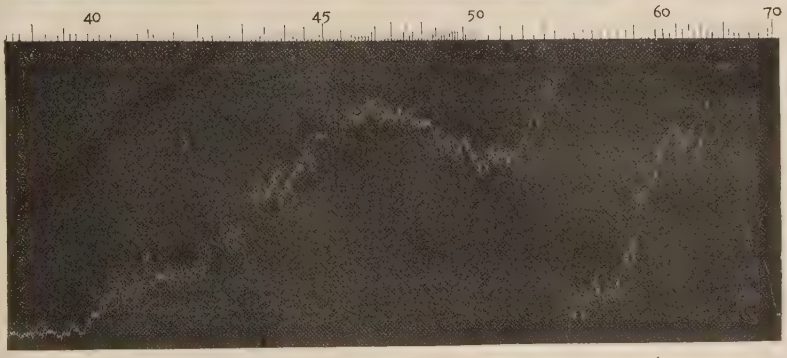
No.	Name.	Plate.	Spectral Class.	
			H.D.: M.W.	Suggested.
7	α UMa.	99/6	F8: F9, F8	F9
19	α Aur.	94/3	Go: G1, G1	G3
45	β Dra.	143/3	Go: Gop, F8	G5
54	ε Cyg.	153/4	Ko: G7, G7	G7
41	α Boo.	94/5	Ko: Ko, G9	Ko
57	ε Peg.	153/2	Ko: Ko, G4	K4

* Abbott, *The Sun*, p. 69.

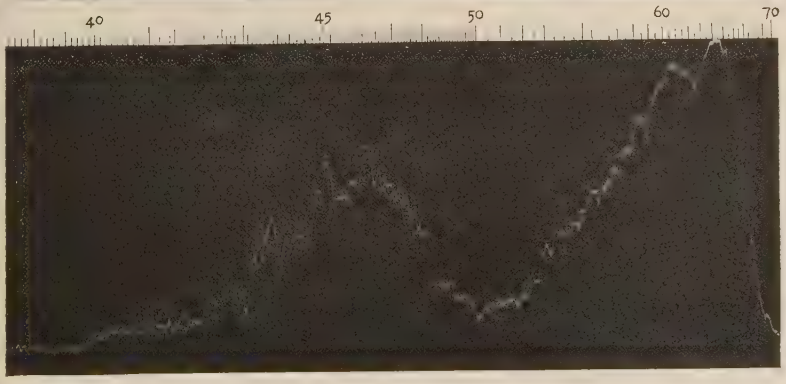

40
45
50
60
70
Polaris
F 9
40
45
50
60
70
Capella
G 3
40
45
50
60
70
Draconis
G 5



ε Cygni
G 7



Arcturus
K 0



ε Pegasi
K 4

Some of the general differences of character in these records are attributable to the difference of the plates employed, but I think it will be considered that, in point of representing the progressive develop-

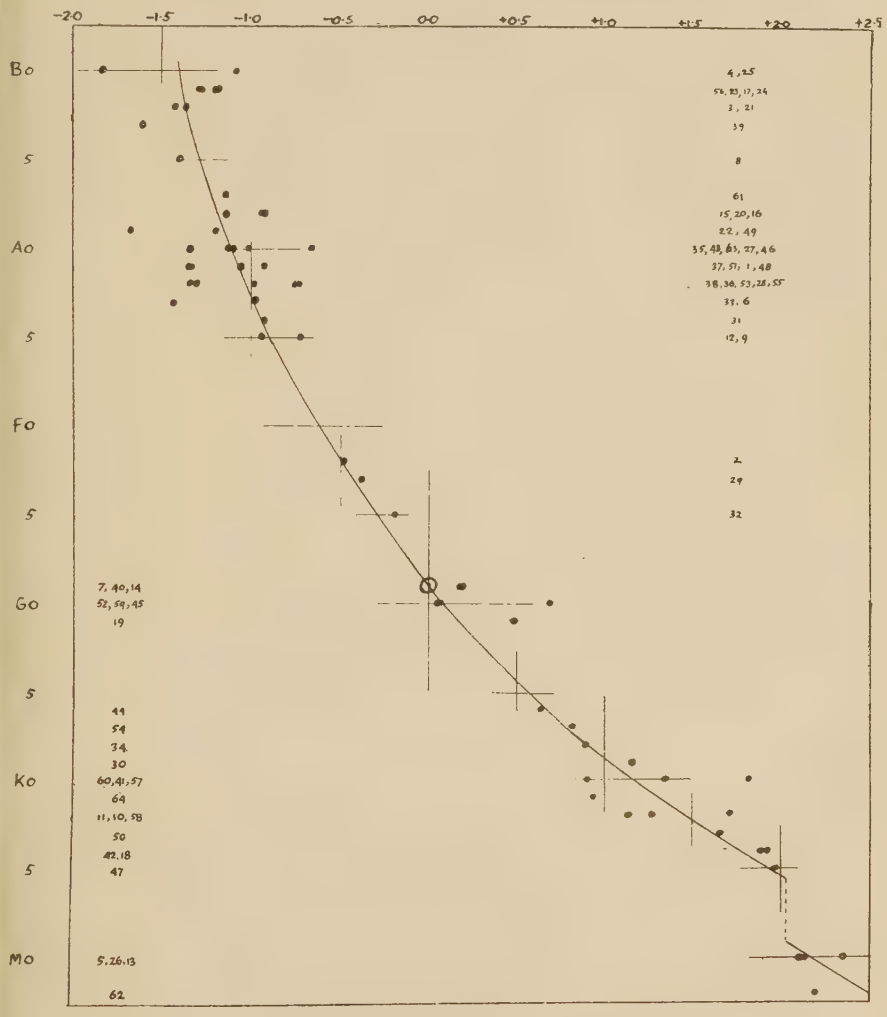


FIG. 3.—Gradient and Mount Wilson Spectrum.

ment of the lines, the suggested class is in no way inferior to any of the accepted ones.

A point that emerges on examination of individual cases is the effect of large zenith distance. It is to be expected that such stars should appear with an excess of red and it will be noticed, for example,

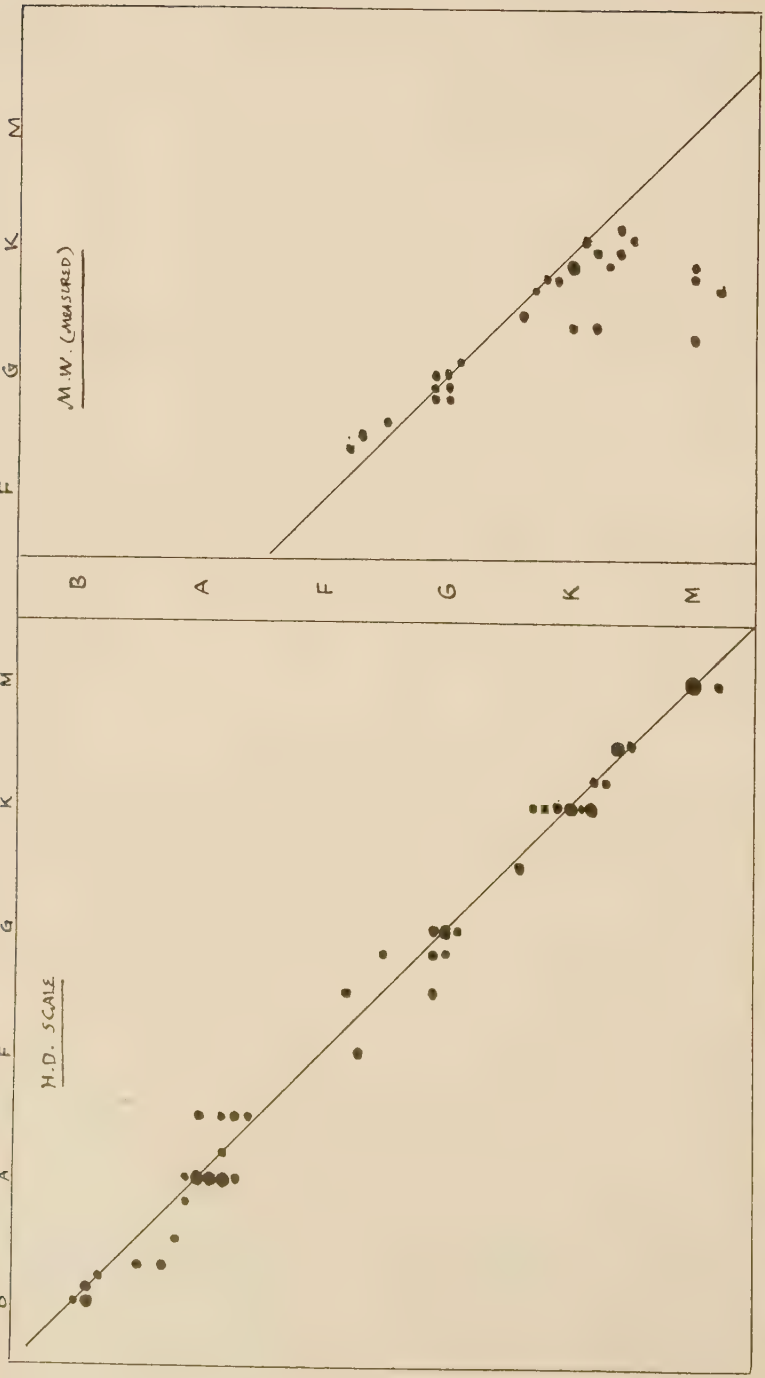


FIG. 4.—Comparison of M. W. Scale (ordinate) with H.D. Scale and M. W. Scale (measured). *N.B.*—The larger dots indicate repetitions.

that (20) *Rigel*, Z.D. 67°; (25) *ζ Ori.*, Z.D. 72°; (28) *Sirius*, Z.D. 76°; and (46) *α Oph.*, Z.D. 67° all lie in marked degree on the red side of the standard line. The matter is complicated by other uncertainties; for example, No. 46, *α Oph.*, is classed A5 by H.D. and Ao by M.W. But, admitting that a considerable dispersion must be expected, the following figure, which shows the distance from standard place for reputed spectral class, plotted against Z.D., confirms a general trend from the blue towards the red, from the zenith downwards.

It will be seen that some of the most anomalous cases in this figure may be traced to the material. Thus (22) *β Tau.* figures only upon Plate 93, in which the photogram of Polaris is known to be anomalous, and the same plate affects (24) *ε Ori.* in the same sense. Others, again, (14) *γ Per.*, (16) *η Tau.*, and (17) *ε Per.*, all appear on Plate 129, and

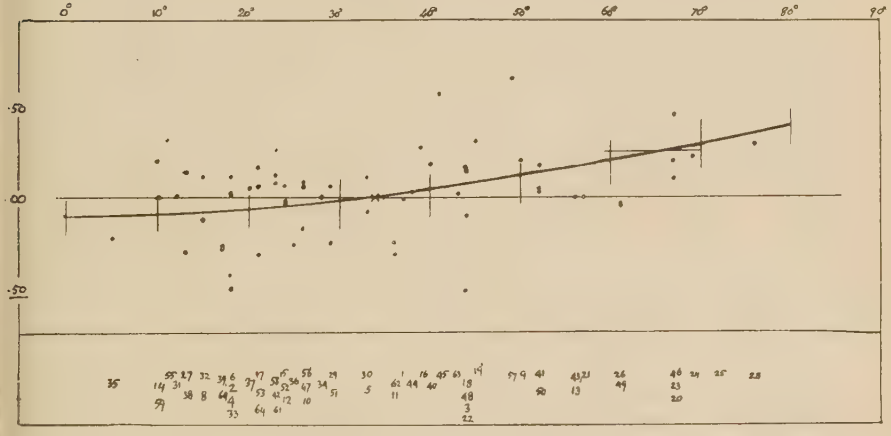


Fig. 5.—*Abscissæ* Z.D. *Ordinate:* Observed Gradient *minus* Standard Curve. Zenith Distance Effect.

are outliers on the blue side. In this case, however, there is nothing to render the Polaris comparison suspect. The matter of varying atmospheric effect is one that will require very careful determination hereafter. I have judged, however, that the indicated effect is sufficiently real to justify application of a correction to the observed gradients before using them for reading the temperatures. The correction applied is represented by the continuous line in the figure, and for purposes of calculation may be written $-0{\cdot}6(1-\cos \text{Z.D.})$, expressed in units of the gradient, and relative to the zenith.

I have shown in fig. 6 the effect of this correction upon the plotting with respect to spectral class. As may be foreseen from the dispersion of fig. 3, the improvement is slight, but it is general and clearly perceptible.

In reference to these comparisons with spectral class we may revert to the question whether the temperature sequence is or is not identical with the spectral sequence. As I have pointed out, there are at present

too many uncertainties in the determination of both of these sequences to allow a final answer. As classifications, however, there seems very little reason for distinguishing them from one another. There are no

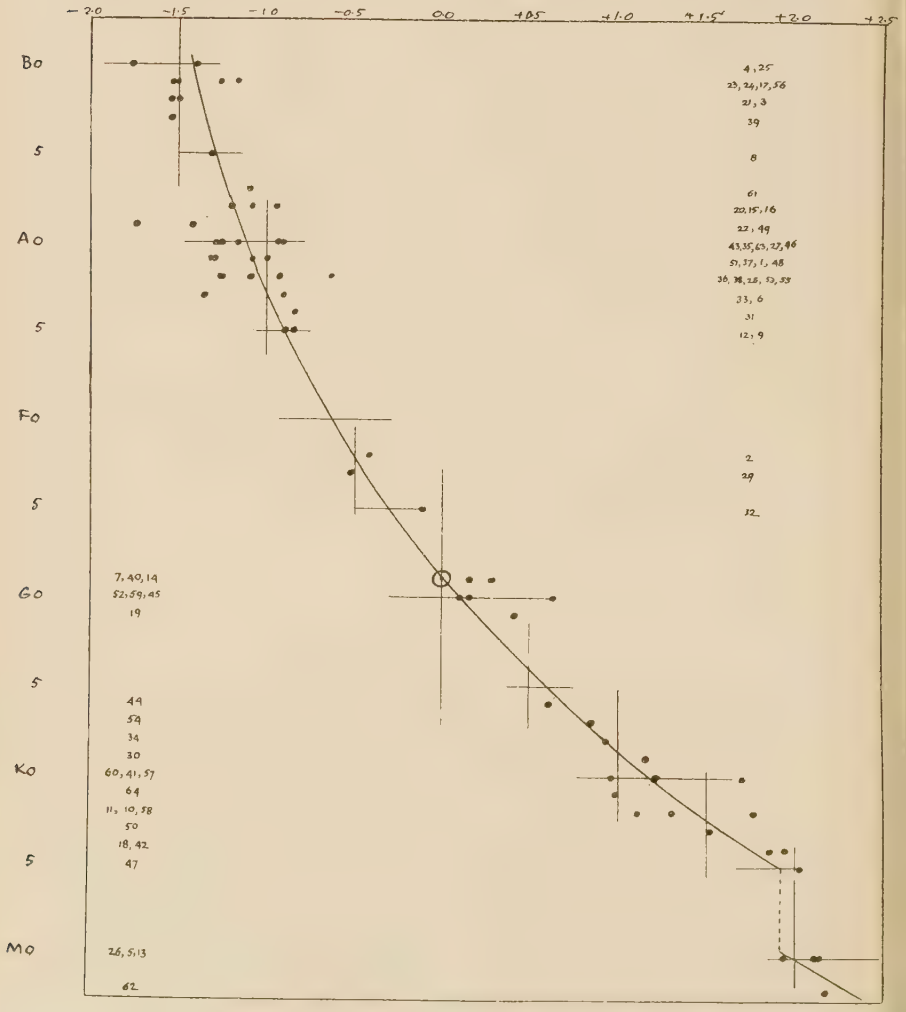


FIG. 6.—Gradient corrected for Z.D. and Mount Wilson Spectrum.

doubt individual peculiarities in the radiation curve of different stars, as there exist spectral peculiarities. A few repetitions of these measures is required in order to establish how far any that are here shown, *e.g.* No. 33, *β UMa.*, and some others that will be referred to below, are inherent or accidental. When any such peculiarities are established, they will invite special attention. In addition to the question of

individual peculiarities, there is the systematic question of a more searching adjustment of the pressure feature (p. 217) to observed results, especially in the discrimination between stars of greater and smaller luminosity. This question must also be reserved. But in the meantime it seems fair to say that temperature is the single dominating feature, and the spectral classification by lines should be made subordinate to it, subject to the recognition of individual vagaries. The temperature classification has this further advantage over the consideration of the spectral lines, that only one single factor enters into it through the whole sequence of change, whereas for the lines we have to alter from one element or characteristic to another several times. This is the reason why inconsistencies still remain in the classification by lines, in spite of the attention that it has received, whereas the present paper shows that even at this stage the whole body of temperature determinations combine to establish a unique standard progression.

Therefore I suggest it would be an advantage to adopt the temperature classification of stars as the standard for fundamental reference; in other words, if the terms Bo, A5, etc., are to carry a single definite meaning, they should be taken for a star with an assigned value of the pressure feature, ψ, and should then connote a temperature as given in the annexed table—subject, of course, to such modification of the numbers as further experience may impose.

TABLE VI.

Bo	25,000°	F6	7,200°
2	20,000	8	6,700
4	17,300	Go	6,200
6	15,600	2	5,800
8	14,300	4	5,300
Ao	13,100	6	4,900
2	12,100	8	4,500
4	11,200	Ko	4,200
6	10,300	2	3,900
8	9,500	4	3,600
Fo	8,900	5	3,500
2	8,300	Mo	3,400
4	7,700	2	3,200

A means of judging the results which is independent of spectral classification is offered by Hertzsprung's "Mean Colour Equivalents of 734 Stars" (*Ann. Sterrewacht te Leiden*, **14**, 1, 1922). In this collection a great variety of sources were employed with appropriate weighting, including mere colour estimations, and these were reduced to a single system by means of the stars in each collection which were to be found also in Wilsing's 'Effektive Temperaturen von 199 Sternen" (*Potsdam*

Astrophys. Obs., No. 74, 1919). The results, expressed as c_2/T, are given in column 9 of the General List. As may be seen from Table I.,

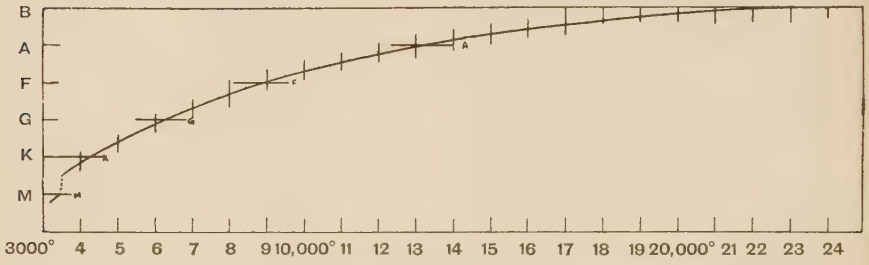


FIG. 7.—Spectral Class and Temperature.

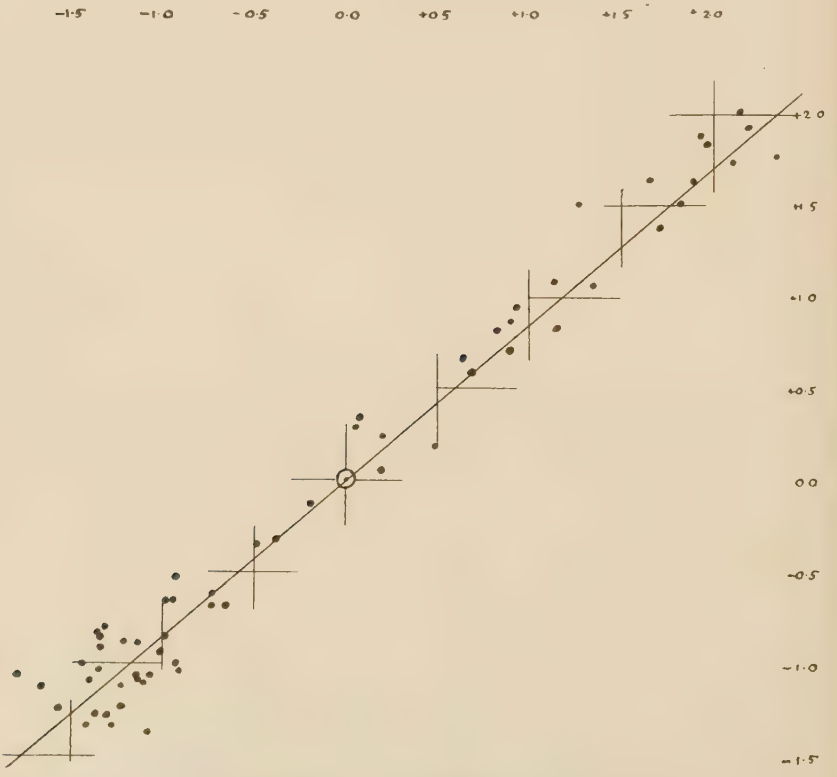


FIG. 8.—Comparison with Hertzsprung's Determinations.

Abscissa: Gradient. *Ordinate:* Hertzsprung's Colour Indices relative to Polaris.

p. 218, this quantity $[c_2/T_{(star)} - c_2/T_{(Polaris)}]$ does not tally precisely with the gradient which I employ. But, in view of the heterogeneous

character of Hertzsprung's material, a refined comparison cannot be aimed at. The annexed figure shows the result obtained by plotting column 8 as ordinate against column 9 as abscissa.

I think this comparison must be considered very satisfactory. The only systematic feature presented is a slightly less range in Hertzsprung's numbers compared to mine, in the ratio 0·86 : 1·00, and this is probably to be explained by his reduction to Wilsing as a standard, for Wilsing's range of temperatures has generally been pronounced too restricted.

A comparison may also be made between the temperatures assigned in my previous paper and those now given. The stars common to the two lists are the following :—

TABLE VII.

No.	Star.	Spectrum.	[1923].	Present Paper.
4	γ Cas.	B0*p* :	16,900°	[30,000]°
63	α Peg.	A0 :	15,000	14,200
1	α And.	A0*p* : A1*n*	15,500	12,600
48	α Lyr.	A0 : A1*s*	11,200	11,600
53	α Cyg.	A2*p* : A2*s*	12,500	10,900
55	α Cep.	A5 : A2*n*	9,200	8,900
2	β Cas.	F5 : F2, F4	8,800	7,800
7	α U Mi.	F8 : F9, F8	6,800	6,500
52	γ Cyg.	F8*p* : G0*p*, G0	6,600	6,200
19	α Aur.	G0 : G1, G1	[5,500]	5,500
54	ϵ Cyg.	K0 : G7, G7	5,100	4,600
57	ϵ Peg.	K0 : K0, G4	4,200	3,600
5	β And.	M0 : M0, G8	3,500	3,300
62	β Peg.	M3 : M2, G7	3,600	3,200

The adjustments are generally downwards, consequent upon that of Polaris. It may seem, by examining the treatment of Z.D., that the greater part of the adjustment of Polaris is due to the zenith distance correction relative to Capella.

The figures given in 1923 expressly disclaimed any authority, but it now appears that they were very close to the truth.

Concluding this confirmatory examination, consider the individual observations of No. 59, δ Cephei. The range of this variable, from $3^{m}\cdot6$ to $4^{m}\cdot3$, places it rather below the limit convenient for my telescope, but, in view of its special interest, 13 observations were obtained. If the observed gradients—after correction for Z.D., which in this case is small—are read for temperature, and the dates of observation converted into days since the last maximum by the formula $2393375^{d}\cdot45 + 5^{d}\cdot366427 \times n$, we have the following table, in which "Phase" denotes "Days since Maximum ÷ Period" :—

TABLE VIII.

Observations of δ Cephei.

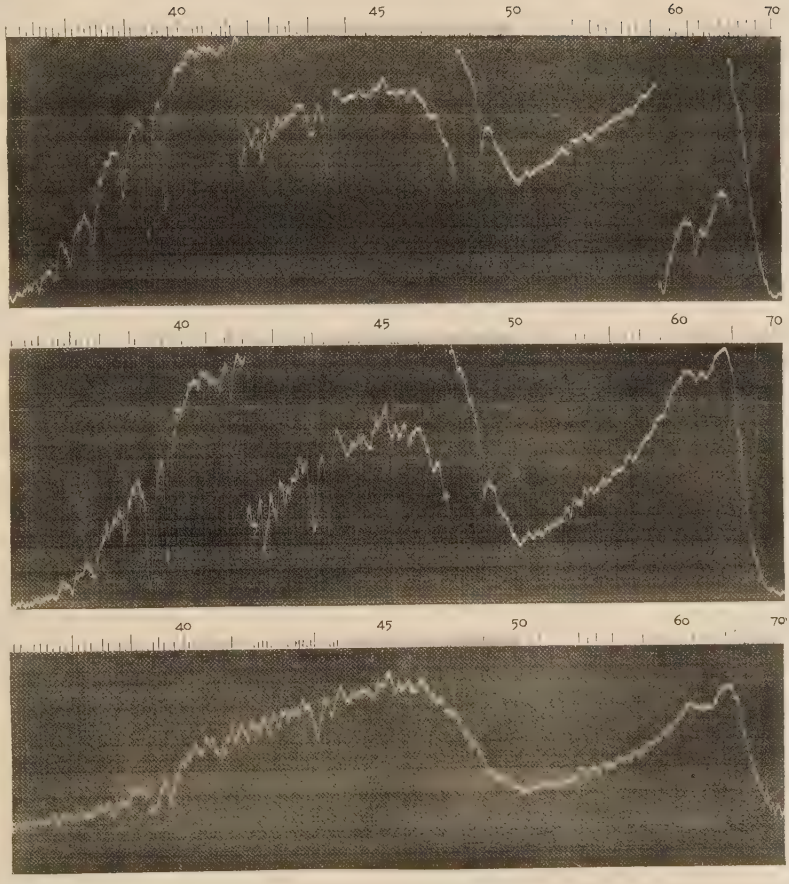
Plate.	T.	Days since Max.	Phase.
	°	d	
108	6500	2·1	·39
109	6450	0·7	·13
123	5780	3·9	·73
125	5900	2·5	·47
127	6510	0·1	·02
128	6370	2·1	·39
130	5900	4·1	·76
131	5780	2·8	·52
132	6400	1·4	·26
150	6510	0·5	·10
151	6020	2·1	·39
154	7800	5·1	·95
155	6980	0·0	·00

Assuming for the moment that the star's emission varies as T^4, we may plot a comparison of 4 log T against the known light curve, 0·4 of a unit of the former being equal to one unit in magnitude. We then obtain fig. 8.

The agreement seems as good as could be looked for, since it must be pretty wide of the truth to suppose that such a star should radiate as a black body.* We may illustrate the way in which these photograms show immediately the character of a spectrum by reproducing three that give respectively the maximum, the minimum, and an intermediate character (Plate 11). The change of features in the Cepheid variables is described very exactly by Adams and Joy (*Proc. Nat. Acad. Sci.*, **4** (1918), 129), from which may be quoted: "The variation in spectral type among the Cepheid variables is mainly a variation in the intensity of the hydrogen lines"; and, with respect to the general spectrum, the most important differences are: "(1) a shift of the maximum of the continuous spectrum toward shorter wave-lengths at maximum of light, a result found by Albrecht; (2) a general slight widening of the spectral lines at minimum; (3) an increase in intensity of the so-called 'enhanced' lines at maximum."

For δ Cephei the hydrogen lines were found by Adams and Joy to vary from type F2 near maximum to F9 at minimum, while the general spectrum varied concurrently from F9 to G0 only. It would appear that occasionally, at any rate, the type of the hydrogen lines carries the general radiation with it, for the temperature 7800° read for the photogram 154/2 corresponds to about F2, but the exceptional position of this reading on the figure confirms the general truth of Adams' and Joy's summary.

* *Cf.* W. Gyllenberg, "Ueber die Intensitätsverteilung im Spektrum des δ-Cephei-Veränderlichen S Sagittae," *Medd. Lund. Astron. Obs.*, serie ii., No. 24 (1919).



Typical Photograms of δ Cephei.

The consideration of δ Cephei raises the question of how far black-body radiation can be inferred as a normal type from these photograms. It is remarked above that when the observations are charted as described, the comparisons for two black bodies should lie upon a smooth line nearly straight. Examining the charts from this point of view, over all the confirmation is remarkably close. I give as illustrations the charts for No. 21, γ *Ori.*, B2, Plate 88/3; No. 12, β *Tri.*, A5, Plate 128/5; No. 41, α *Boo.*, K0, Plate 94/4; and No. 5, β *And.*, M0, Plate 101/4.

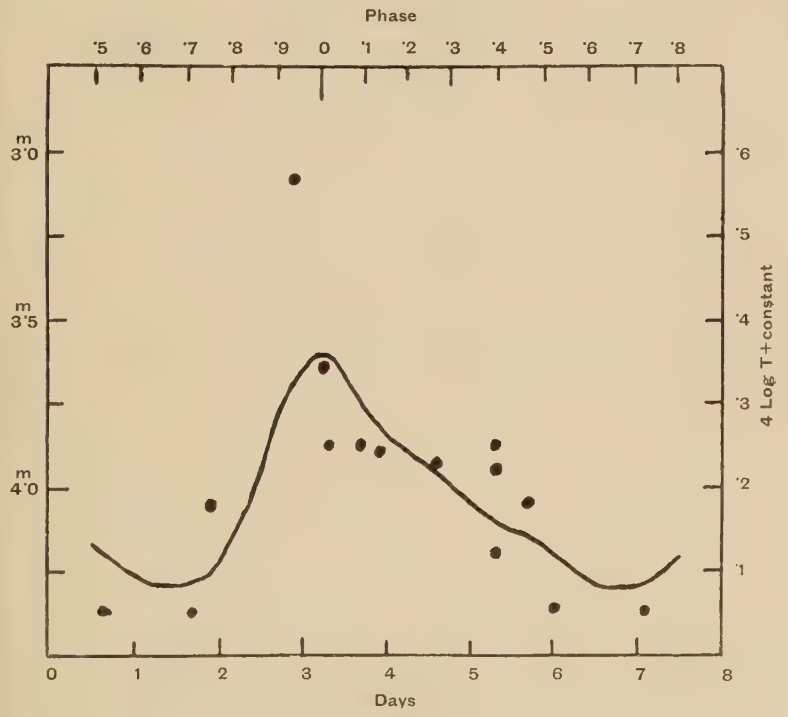


Fig. 9.—δ Cephei.

Particularly it is remarkable how the wild oscillations of the banded spectrum of an M-type star, superposed to the equally marked sensitivity of the photographic plate, disappear completely. The irregularities that remain are shown by other charts of the same stars to be for the most part accidental faults. One exception to this may be noted as of frequent occurrence. For the comparison with the early-type stars the measures for the shorter wave-length lie above the line, for the later types they lie below. It is known that the solar spectrum is deficient in the ultra-violet. If this were a progressive quality showing in a B or A star compared to Polaris, and in Polaris compared to a K or M star, we should have such a feature as is shown in the charts.

The charts shown are fairly characteristic of the mass, and I con-

clude that the normal star is not inadequately represented over the whole spectral range by a black radiation, qualified, as explained on p. 217, by a progressive reddening of the disc towards its margin, which is the atmospheric effect of the star. The feature referred to above suggests that this allowance for reddening of the disc requires to be increased.

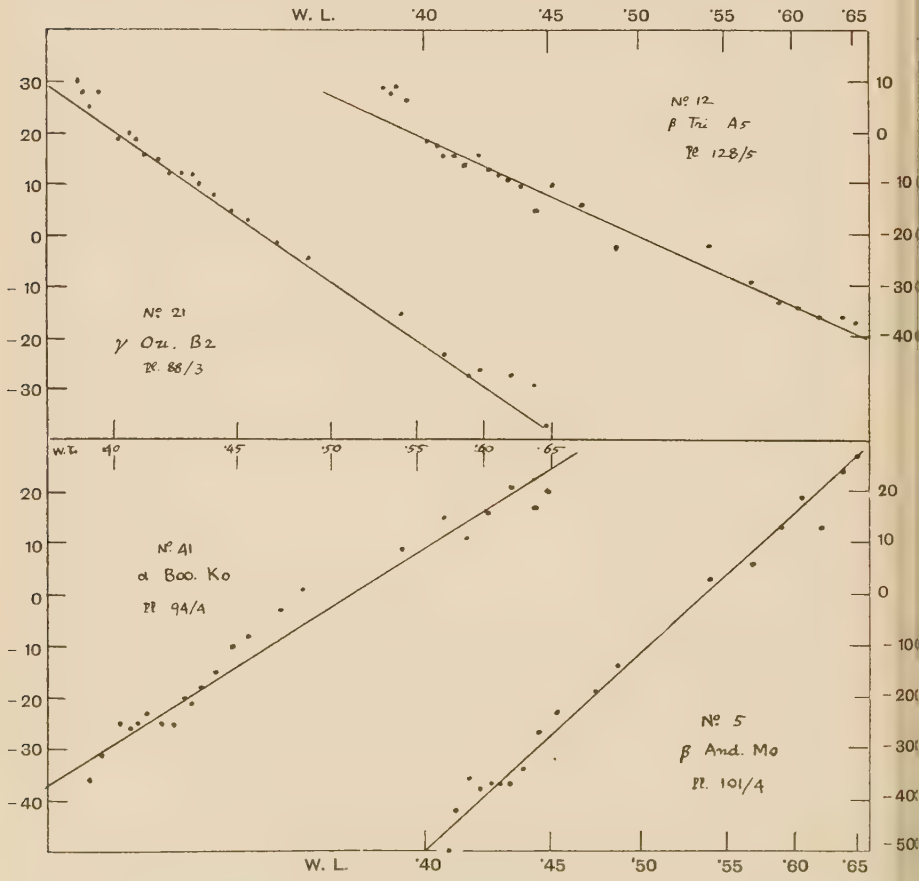


FIG. 10.—Normal Charts.

There are, however, other cases of inherent difficulty, when the chart cannot be held to represent a straight line. A conclusive consideration of these must be deferred until material is collected that shows how far their peculiarities are certain, but one or two may be mentioned in illustration. One of these is No. 4, γ *Cas.*, B*op*. The photogram of this star was shown in my previous paper (Plate 9), and reference was made to the very low temperatures ascribed to it by Wilsing and H. H. Plaskett (p. 203). At that stage my photograms were unable to show any

measures in the red. The present series shows the red as a singular feature, the chart being two straight lines. The spectrum is, in fact, a composite one. The star Boss P.G.C., No. 201, $5^{M}\cdot8$, Ao, is distant from γ *Cas.* by $\Delta\alpha = +5^{s}$, $\Delta\delta = -22'$. In any trail in R.A. the spectra are superposed. I have a photogram specially taken which shows the hydrogen series of the second star superposed to the spectrum of γ *Cas.*,

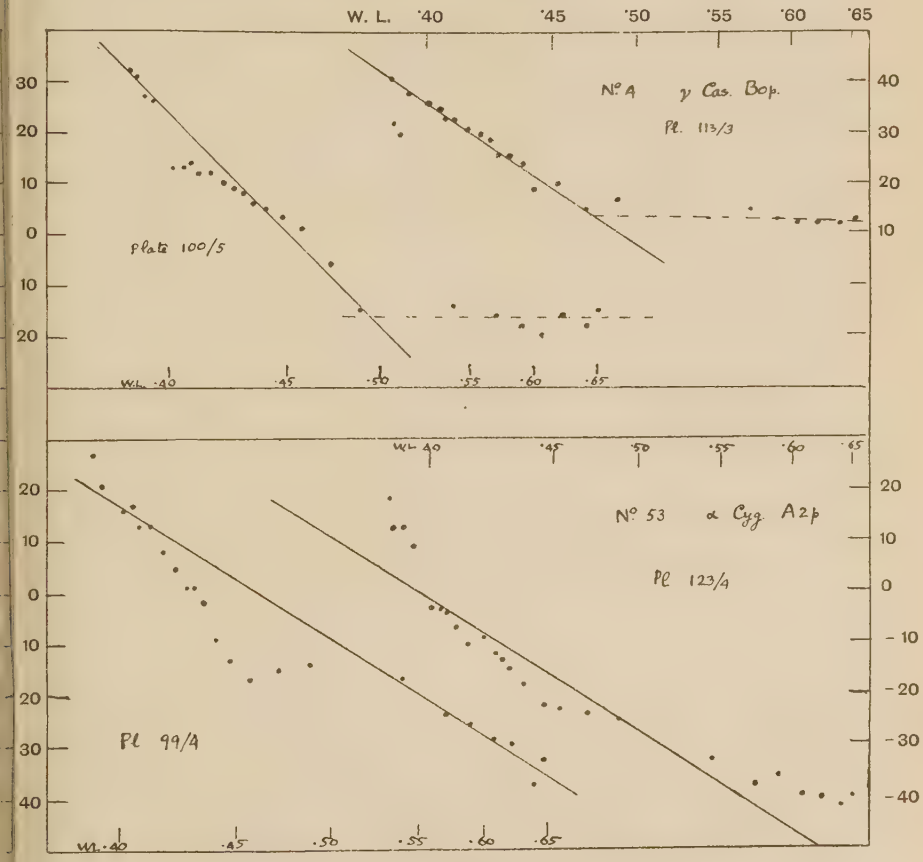


Fig. 11.—Abnormal Charts.

the bright H_a of the latter falling between H_γ and H_δ of the former. This makes the reading very uncertain. Probably the gradients I have taken, disregarding the red altogether, are too high, and with them the associated temperature.

There are, however, some cases where no external explanation is forthcoming, and the peculiarity seems to arise from the star's own peculiar radiation. The δ Cephei series offers some examples. The charts of No. 53, α *Cyg.*, A2*p*, give others. These are shown in the

figure. Comparing these with the whole remaining body of charts, I am unable to consider them accidental. But it is desirable to accumulate more material on both regular and abnormal charts before drawing any conclusions.

In the foregoing discussion the stars of high luminosity form the great majority, and therefore may be considered as determining the standard spectral curve. The following stars of absolute magnitude $> 1^{M}\cdot5$ are, however, found in the list:—No. 2, β *Cas.*, $1^{M}\cdot6$, F2; No. 6, δ *Cas.*, $2^{M}\cdot0$, A3; No. 29, α *CMi.*, $3^{M}\cdot2$, F3; No. 30, β *Gem.*, $1^{M}\cdot7$, G9; No. 31, ι *UMa.*, $2^{M}\cdot2$, A4; No. 32, θ *UMa.*, $3^{M}\cdot0$, F5; No. 40, η *Boo.*, $2^{M}\cdot8$, F9; No. 64, γ *Cep.*, $2^{M}\cdot3$, K1.

By examining figs. 3, 6, pp. 233, 236, it will be seen that all of these eight stars lie to the right or red side of the standard curve, except No. 29, which has been moved across the line by the Z.D. correction, and No. 64. The amounts are of the order of ·10 in gradient, or, say, 200° for a star of type Go. Their regularity forbids us to call this small quantity accidental.

For comparison take also the following stars of very great luminosity:—No. 3, γ *Peg.*, $2^{M}\cdot9$, B2; No. 7, α *UMi.*, $3^{M}\cdot0$, F9; No. 17, ϵ *Per.*, $2^{M}\cdot0$, B1; No. 21, γ *Ori.*, $2^{M}\cdot0$, B2; No. 23, δ *Ori.*, $2^{M}\cdot4$, B1; No. 24, ϵ *Ori.*, $2^{M}\cdot4$, B1; No. 25, ζ *Ori.*, $2^{M}\cdot8$, Bo; No. 26, α *Ori.*, $3^{M}\cdot8$, Mo; No. 45, β *Dra.*, $3^{M}\cdot5$, Go; No. 52, γ *Cyg.*, $3^{M}\cdot0$, Go; No. 56, β *Cep.*, $2^{M}\cdot3$, B1; No. 59, δ *Cep.*, $2^{M}\cdot7$, Go. Of these 12 stars, 7 are of type B, but we need not distinguish them from the others. The standard line has been drawn through No. 7, α *UMi.* It will be found that the whole of the members of the list lie on or to the left or blue side of the standard curve, excepting No. 17, ϵ *Per.*, No. 45, β *Dra.*, and No. 56, β *Cep.* I have shown reason above for suggesting that No. 45, β *Dra.*, is incorrectly classed.

Until the exact position of individual stars is established more certainly, both as to spectral class and to temperature, it is not desirable to draw detailed conclusions. It will be noted, however, that though the cases referred to do not include any stars of really low luminosity, they range over a considerable span, viz. $7^{M}\cdot3$, or more than 800-fold in brightness. It will also be remarked that they conflict with a view commonly accepted, that for the same spectral class the effective temperature of a giant star is lower than that of a dwarf. I would only remark here that the evidence for that view is drawn from spectrograms which certainly show very varying extension into the ultra-violet, but in the other direction do not go beyond H_{β}.* My experience in reading photograms leads me to say that any conclusion as to gradient and temperature which is confined to that end of the spectrum alone is very uncertain. I would further suggest that the observations in question indicate some complete difference of atmospheric state, such as δ Cephei develops between its maximum and minimum, and should be treated as belonging to the pressure effect, rather than anything that should rightly be called a temperature effect.

This consideration leads me to add a word with regard to the term

* *Cf.* Adams, *Ap. J.*, 39, 89 (1914).

"effective temperature." The form for measuring the intensity of radiation of a star, consisting of a pure temperature factor, qualified by a pressure factor, was adopted on trial (p. 217), taking the Sun's case as model. To the best judgment I can form after trial, if we set aside a small number of cases, like δ Cephei and perhaps like α Cygni, this solar model requires very little modification even for a range of stars that must differ enormously from the Sun and also from one another. Attention should then be addressed to assigning the pressure factor more precisely and giving it a clear physical meaning. It seems to me that the onus of variation from type may properly be thrown upon it. If so, the suspensory word "effective" would disappear, and the temperature of the star, as determined above, would be a term sufficient in itself.

On the Mass-Ratio in Double Stars. By G. Shajn.

(*Communicated by the Secretaries.*)

1. Eddington's theory of radiative equilibrium gives a relation between the luminosity and the mass of a star. In reality these formulæ give not the luminosities themselves, when the mass and effective temperature are known, but only the difference of absolute magnitudes of two stars according to $\frac{M_1}{M_2}$ and $\frac{T_1}{T_2}$. The results obtained by Eddington are in conflict with the previously accepted theory of stellar evolution, and call in question the evolutionary sense of Russell's diagram. If the mass remains constant the diagram cannot be interpreted as indicating the course of evolution of an individual star. According to the most recent results of the theory of radiative equilibrium, density does not play an essential part in Russell's diagram. The low luminosity of the stars of the descending branch is fully accounted for by their comparatively small mass without appeal to any other physical difference (*M.N.*, 84, 332).

Thus two questions arise: (1) Can the statistical diagram be interpreted as the course of evolution of an individual star, or does it indicate something different (for instance, locus of equilibrium . . .)? (2) If it is really the course of evolution, what is its cause—decrease of mass, deviation from a perfect gas, or something else? I think that the consideration of physically connected double stars may be of use for discussing this subject.* The importance of double stars in this question is evident, for we can with some probability speak of common origin of the components of a double star. Another important factor is

* This paper was nearly ready when there appeared that of Vogt (*Zeitschrift für Physik*, 1924 August, 2) discussing the same subject—it will be mentioned later. I consider the question in a somewhat different manner, and come to somewhat different results.

that the difference of absolute magnitudes of both components is equal to that of the apparent ones ($\Delta M = \Delta m$). Thus it can be written in the form

$$2{\cdot}512^{-\Delta m} = \left(\frac{M_1}{M_2}\right)^{\frac{1}{3}} \left(\frac{1-\beta_1}{1-\beta_2}\right)^{\frac{2}{3}} \left(\frac{T_1}{T_2}\right)^{\frac{4}{3}}.$$

Evidently it does not enable us to determine $\frac{M_1}{M_2}$ from Δm.

A table was computed by the formula (1) (*M.N.*, **84**, 309) to give a series of values $1-\beta$ as a function of the mass; and another table to give $\frac{1-\beta_1}{1-\beta_2}$ as a function of $\frac{M_1}{M_2}$. Assuming for M_1 definite values, such as 0·5, 1·0, 2·0, . . . and arbitrary ones for M_2, it is possible to compute and to draw curves for each value of M_1, the abscissæ being Δm and ordinates $\frac{M_1}{M_2}$ (essentially like that given by Eddington, *M.N.*, **84**, plate 8). In computing these curves the coefficient in (1) (*M.N.*, **84**, 309) was taken to be 0·00260, as the computations were made before the publication of Eddington's last paper. But this difference will not affect the result.

2. In most cases it is easy to discern the giant systems from the dwarfs from the spectra of the two components. In giant systems the spectrum of the secondary is nearly always of an earlier type, and in dwarf systems of a later type than that of the primary (*cf.* the investigations of Lane, Leonard,* Doig, Abetti, and my own unpublished observations by Seares' method of exposure-ratios of 114 double stars). If one of the spectra belongs to the type from B_0 to F_2, it may be possible to estimate the absolute magnitude of the components (Adams and Joy, *Ap. J.*, **56**, 4; **57**, 5). On account of both these criterions and the proper motion, the absolute magnitude of the secondary can be nearly always approximatively determined. The corresponding approximate mass can be taken out of the tables of Eddington or Seares. A considerable error in L and M gives a very small one in $\frac{M_1}{M_2}$. An error of $\pm 1{\cdot}0$ in the absolute magnitude corresponds to a maximum error of 50 per cent. in $\frac{M_1}{M_2}$, while for $\Delta m < 3^m{\cdot}0$ it scarcely reaches 15 per cent. This may allow us to compute with some accuracy $\frac{M_1}{M_2}$ for systems whose absolute magnitude is known only indirectly. The observational material can thus be augmented nearly four times. Vogt used only 93 pairs with known absolute magnitude, but for some of these the luminosity is rather imperfectly known. It may be said that the systems with luminosity found indirectly give results with an accuracy differing little from that which is got from systems with directly determined absolute magnitude. Such a determination for a great number of double stars

* Leonard, *Lick Obs. Bull.*, No. 343, the most extensive investigation.

(342) has some interest also from another point of view. In the computation Δm was reduced to the bolometric scale and a correction for temperature was introduced. The application of the theory of radiative equilibrium to the giants has little uncertainty. Eddington has pointed out the possibility of its application to the dwarfs.

3. The differential formula $\frac{dM}{M} = -3\frac{d\pi}{\pi}$ shows that an error of 20 per cent. in the parallax gives an error of 60 per cent. in the mass of a double star. It is still more difficult to determine the mass-ratio of the components. Nearly all the ratios known refer to the dwarfs. It is of interest to compare the observed ratios with the theoretical ones.*

TABLE I.

	$\frac{M_1}{M_2}$ (computed).	$\frac{M_1}{M_2}$ (observed).	O-C.		$\frac{M_1}{M_2}$ (computed).	$\frac{M_1}{M_2}$ (observed).	O-C.
η Cassiop.	0·55	0·76	+0·21	ζ Herculis	0·41	0·43	+·02
Sirius	0·13	0·29	+0·16	Krueger 60	0·76	·56	−·20
ϵ Hydræ	0·64	0·9	+·26	70 Ophiuchi	·79	·82	+·03
η Cancri	0·95	1·00	+·05	α Aurigæ	·86	·79	−·07
ξ Urs. maj.	0·90	1·00	+·10	β Aurigæ	·98	1·00	+·02
γ Virginis	0·99	1·00	+·01	80 Tauri	·31	·39	+·08
α Centauri	0·92	0·85	−·07	μ Herculis	·89	1·0	+·11
ξ Bootis	0·81	0·87	+·06	o_2 Eridani	·77	1·0	+·23

The agreement is satisfactory. The average $O-C = \pm 0{\cdot}10$ can be explained by the errors of observations. The differences are somewhat systematic, but will be less for $\mu < 2{\cdot}1$.† The stars 85 Pegasi, 9 Argus, τ Cygni were not compared, as the observed values of $\frac{M_1}{M_2}$ are very uncertain. From this list was excluded the companion of Procyon (see Eddington, *M.N.*, 84, 314). A sort of independent proof of the possibility of applying the theory to the dwarfs is given by this more or less satisfactory agreement.

4. Such computations were made for 342 systems. Only a few of them can be optical ones. The criteria are: orbital motion, common motion of both components, their immobility when the distance is less than 30″ (in exceptional cases the distance is greater). I have considered 238 double stars,‡ collected by Leonard; 58 supplementary ones, selected from the H.D. (their spectra are known with a little less accuracy than these of the first group); and 46 double stars with unknown spectrum, which I observed by Seares' method of exposure-ratios. Accuracy not less than ½ of a spectral class may be expected for the latter (the spectral class deduced from the colour-equivalent). These

* The sources are Aitken's binary stars, Eddington (*M.N.*, 84, No. 5), and Hertzsprung (*B.A.N.*, ii., No. 43).

† In the computations the value $\mu = 2{\cdot}1$ is used.

‡ Including some binary stars from *M.N.*, 84, 312–313.

data, when plotted on a diagram (with $\frac{M_1}{M_2}$ as abscissæ, and Δ Sp. as ordinates) show distinctly that Δ Sp. increases with decreasing values of $\frac{M_1}{M_2}$, *i.e.* that the greater the difference of the masses of the components the more their physical properties (spectra) are different. The average results are given in Table II.

TABLE II.

Giants.				Dwarfs.				Systems with B-type Component.			
Δ Sp.	$\frac{M_1}{M_2}$.	*n.*	Limits of Δ Sp.	Δ Sp.	$\frac{M_1}{M_2}$.	*n.*	Limits of Δ Sp.	Δ Sp.	$\frac{M_1}{M_2}$.	*n.*	Limits of Δ Sp.
0·07	0·88	78	0·0 – 0·4	0·12	0·88	65	0·0 – 0·4	0·12	0·68	38	0·0 – 0·4
0·75	·72	12	0·5 – 0·9	0·67	·85	16	0·5 – 0·9	0·57	·41	10	0·5 – 0·9
1·18	·66	33	1·0 – 1·4	1·17	·70	15	1·0 – 1·4	1·13	·34	3	1·0 – 1·4
1·62	·62	20	1·5 – 1·9	1·74	·63	12	1·5 – 1·9				
2·15	·45	18	2·0 – 2·4	2·90	·35	4	2·0 – 4·5				
2·64	·56	9	2·5 – 2·9								
3·44	·37	9	3·0 + 4·5								

This is a new interpretation of the fact that Δ Sp. increases with Δm; the reality of this relation is beyond doubt. This fact, along with the result mentioned above that in giant systems the spectrum of the companion belongs to an earlier class (nearly always) than that of the primary, while in dwarf system the spectrum of the secondary is of a later type, leads us to conclude that both these relations indicate the course of stellar evolution. It is very difficult, and perhaps even impossible, to see in these facts anything other than the evolution of an individual star according to Russell's theory. Moreover, it is evident that the spectrum results from the physical condition in the course of stellar evolution. Double stars show that Russell's diagram can be only interpreted as the course of evolution of a single star.

5. I will point out some other results. It must be said that systems with both components of B-type differ sensibly from others (Table II.). Even a considerable difference of masses, or a large value of Δm, does not produce any sensible difference in the physical properties of the components. Although the mass-ratio decreases also for these stars as Δ Sp. increases, the comparison with the other classes of stars shows a great difference. Within the limits of Class B are involved stars with very different masses. This fact indicates the high limit of absolute magnitude for the B-type stars; there are no indications that increase in absolute magnitude must necessarily be in direction B_0, $B_1 \ldots B_9$.

6. The data of Table II., plotted on a graph, show that, the values of Δ Sp. remaining unchanged, the mass-ratio remains more or less equal for giant and dwarf systems. As concerns the relation between Δ Sp. and Δm, the latter will be a little greater for dwarfs than for giants, the value of Δ Sp. being the same. It is true that as long as the application

of the theory of radiative equilibrium to the dwarfs may be considered as a legitimate one, this peculiarity, which may be brought into agreement with the Russell-Hertzsprung theory, requires further consideration. There is no possibility, however, of deciding if this fact is the result of a peculiar selection of stars (the influence of the selection is undoubtedly of great importance) or if it has a real foundation.

7. Aitken* has brought together in a table the mass-ratios of spectroscopic binaries, and shows that the mass-ratio increases with the advancing of types from B to G. The material is not sufficiently extensive and is selected. It is of interest that the same relation is found again, if such a table is derived from visual double stars, without discrimination of giants and dwarfs (Table III.). The principal cause of it is the smallness of the mass-ratio of B-type stars (compare with Table I.); yet the influence of selection of stars is too great to permit of any conclusion.

TABLE III.

	$\frac{M_1}{M_2}$ (Double Stars.)	*n.*		$\frac{M_1}{M_2}$ (Spectr. Bin.)*	*n.*
$B_0 - B_9$	0·60	50	$B_0 - B_8$	0·70	11
$A_0 - A_9$	·77	69	$B_9 - A_5$	·75	16
$F_0 - F_9$	·78	109	F	·92	3

8. We may think at first that double stars indicate a gradual decrease of mass during their lifetime. Such is the conclusion of Vogt (*Zeitsch. f. Physik*, 1924, 2). Vogt gives a table for 93 double stars, where the mass-ratio is given as a function of the spectrum of the primary. When such a table was computed for 342 double stars, it was found that the character remained essentially the same, although the number of pairs was nearly four times greater. The increase of $\frac{M_1}{M_2}$ when advancing along types M, K . . . A . . . K, M is seen (Table IV.) in a more regular manner. If we consider the loss of mass as proportional to the radiation, and thus to the mass itself, it is natural to expect the increase of $\frac{M_1}{M_2}$ of an individual system with the time. Yet the table of Vogt (*Zeitsch. f. Physik*, 1924, 2, 141) and Table IV. in the present paper do not allow, to my mind, any conclusion of a gradual decrease of mass with time. It must be indicated, firstly, that there are double stars with very different Δ Sp. Moreover, *a priori*, we can expect the existence of systems with very different values of $\frac{M_1}{M_2}$. Further, it is natural to think that the masses being equal, the speed and the course of evolution must be the same, while they will be different for unequal masses. This is to some extent confirmed by observations. When a giant system is taken, with the spectrum of the primary M or K, for instance, the spectrum of the companion may be M, K, G . . . A . . . . For a dwarf system with the spectrum K of the primary, that of the secondary can be evidently only of K- or M-type. In this case it is natural to expect

* *The Binary Stars*, 207.

on the average the decrease of Δ Sp. shown in Table IV. The values of Δ Sp. given together with $\frac{M_1}{M_2}$ confirm this supposition. Moreover, as is indicated by Table I., a considerable increase of $\frac{M_1}{M_2}$ may be expected independently of loss of mass during the lifetime of the star. This is the principal cause.

Table IV.

	Δm.	Δ Sp.	$\frac{M_1}{M_2}$.	n.
M	4·43	3·73	0·16	3
K	2·19	2·25	·43	28
G	1·26	1·72	·60	36
F	0·90	0·74	·74	75
B	1·79	·41	·60	50
A	1·24	·51	·77	69
F	0·77	·47	·86	35
G	0·62	·26	·89	32
K	0·63	·19	·90	12
M	1·00	·00	·83	2

The material is selected further from other points of view. We are in very disadvantageous conditions as concerns the study of dwarf systems, because the photographic Δm is larger than the visual (the spectrum of the secondary of later type than that of the primary). Our possibilities being limited, only such dwarf systems are accessible to our investigations whose Δm and consequently Δ Sp. are small. It is quite different for the giant systems, where the photographic value of Δm is smaller than the visual. There are many close pairs which cannot be separated on the plates, and only a considerable Δ Sp. allows the observation of the duplicity of these systems. For giant systems with great Δ Sp. the conditions of selection are very favourable. Thus the selection has a great influence upon the peculiarity that $\frac{M_1}{M_2}$ increases, when we go from giants to dwarfs. It may be said that if Δm would remain the same for a given value of Δ Sp. in all double stars, giants as well as dwarfs, the discussion of the material from the point of view of the theory of radiative equilibrium would bring out the peculiarity shown in Table IV. Firstly, it is seen from the formulæ 1 and 2 (*M.N.*, 84, 309–310) that for the same Δm, $\frac{M_1}{M_2}$ will be far smaller for giants than for dwarfs. For instance, for dwarf and giant systems (abs. mag. of the secondary $+6{\cdot}0$ and $-0{\cdot}8$), having $\Delta m = 2{\cdot}0$, $\frac{M_1}{M_2}$ will be 0·63 and 0·44 respectively. Further, the correction for the temperature term increases this effect, as $T_{second.} > T_{prim.}$ in giant systems and $T_{second.} < T_{prim.}$ in dwarf ones. The reduction to bolometric scale has a smaller effect but still in the same direction. Thus the character of Table IV. partly

depends on the present form of the equations of the theory of radiative equilibrium. But if we content ourselves only with the two peculiarities mentioned above, the influence of selection is so great that the reality of the increase of $\frac{M_1}{M_2}$ with the time can scarcely be accepted. Besides, the spectroscopic binaries, being more homogeneous, do not show this relation (see Table III.). It is very difficult to account for all the circumstances for purposes of getting numerical interpretations of the influence of selection, but the fact that this influence is of the same order as in Table IV. leaves no doubt. For the same reason table ii. of Vogt (*Zeitsch. f. Physik*, 1924, 2, 142) does not confirm the reality of possible decrease of mass with the time more than Table i. of the same paper.

9. The relation between $\frac{M_1}{M_2}$ and Δm (Table IV.) can give some information as to the difference of brightness of both components of a spectroscopic binary in the case when $\frac{M_1}{M_2}$ is known from the observations. Furthermore, this table and equations indicate the conditions under which the spectra of both components may be expected to be seen. Only a small number of spectroscopic binaries with spectra of both components are known. There remains the question: Are the majority of spectroscopic binaries single stars really, whose variations of radial velocities are due to physical processes (pulsation), or have we to deal with systems whose secondary spectrum cannot be brought out owing to the faintness of the companion? From this point of view Table IV. is interesting. It is scarcely possible to observe both spectra when $\Delta m > 1{\cdot}0$ or $1{\cdot}5$.* Both spectra can be expected to be seen only when $\frac{M_1}{M_2} > 0{\cdot}50$. If we judge the mass-ratio from visual binaries and roughly suppose that the influence of selection to be more or less equal in the two cases, the great number of spectroscopic binaries with a single spectrum supports the supposition that the cause of the variability of radial velocity for a part of them is to be attributed to physical causes and not to orbital motion. In the list of spectroscopic binaries with known $\frac{M_1}{M_2}$ † (omitting β Lyræ as a quite peculiar system), $\frac{M_1}{M_2}$ averages $0{\cdot}748$. If, conversely, we compute Δm (the mass of the secondary is known), we find it equal $0^m{\cdot}8$, *i.e.* a value which really should allow us to observe both the spectra. Only for 4 out of 31 systems do we get $\Delta m > 1{\cdot}4$. In order to get $\Delta m < 1{\cdot}4$ for them we must allow their masses to be considerably greater than is probable. The agreement between theory and the data of spectroscopic binaries is, in general, to be considered as satisfactory. Another well-known peculiarity of spectroscopic binaries must be mentioned, namely, that, if the spectra of both components are alike, their masses are generally equal, while in the case of different masses the secondary spectrum is of an earlier type than that of the primary.

* O. Struve, *Ap. J.*, 58, 148. † Aitken, *The Binary Stars.*

As Δm cannot be $>1\cdot5$ the difference of spectra of both components is very important. The photographic Δm = the visual $\Delta m + \Delta$ (Col. Ind.). Though the second term is generally small, we have favourable conditions when Δ (Col. Ind.) is negative, *i.e.* when the spectrum of the primary is of a later type. The relation mentioned above is partly a result of evolution (ascending branch) and is partly due to the favourable conditions for the observation of such systems.

Summary.

In the present article the relation between the mass-ratio of double stars and the spectrum difference of the components is considered in connection with the question of the evolution of the stars.

1. Using the criterion of subdivision of the double stars into giant and dwarf systems (by means of the spectra of the two components) and the relation $\Delta M = \Delta m$, the mass-ratios for 342 systems on the basis of the theory of radiative equilibrium were computed.

2. The comparison of observed values of $\frac{M_1}{M_2}$ (principally for dwarfs) with the theory shows a satisfactory agreement for small values of M. The average O − C is 0·10 (Table I.).

3. The difference of physical properties (spectra) considerably increases with the decrease of the mass-ratio of components (Table II.).

4. Considering the fact that the spectrum of the secondary is of an earlier type in giant systems and of a later one in dwarf systems than that of the primary, Table II. shows that Russell's diagram indicates the course of evolution of an individual star.

5. The double stars of B-type are quite peculiar. Though the Table III. is not selective, there is very high dispersion in masses and absolute magnitudes within the limits of the B-type.

6. When the value of Δ Sp. is the same, the mass-ratio is nearly equal for giant and dwarf systems, while Δm is somewhat greater for the dwarfs.

7. A comparison of the relation between $\frac{M_1}{M_2}$ and the spectrum for the spectroscopic and visual double stars shows some similar and some dissimilar characteristics.

8. The increase of $\frac{M_1}{M_2}$ in double stars, arranged according to the spectrum of the primary (M, K, G . . . A . . .), is chiefly the result of the selection of the stars. This relation does not allow any conclusion about the decrease of the mass with the time.

9. The application of the relation between Δm and $\frac{M_1}{M_2}$ to spectroscopic binaries brings out the conditions for the possibility of observing the spectra of both components, and shows that observations agree in general with theory.

Pulkovo:
1924 *December.*

On Continuous Hydrogen Radiation in Celestial Spectra. By J. W. Nicholson, F.R.S., Fellow of Balliol College, Oxford.

The main object of this note is to direct further attention to an investigation by Dr. W. H. Wright,* who subsequently republished it, with slight alterations, in *Nature*. In our opinion, it is capable of giving, if pursued in certain experimental directions, information which is fundamental for the further generalisation of the quantum theory of emission spectra.

The first discovery of the phenomenon with which it is concerned was probably made by Evershed,† who, in his observations of the eclipse of 1898 January 22, noticed a continuous spectrum belonging to the solar chromosphere and prominences. It began—afterwards extending towards the ultra-violet—so near the head of the Balmer series of hydrogen ($\lambda 3646$) as to excite some suspicion that it might be due to hydrogen. R. W. Wood‡ subsequently found a similar continuous spectrum on the violet side of the sodium series of dark lines. A spectrum very like the one in question is known to exist in planetary nebulæ.§ It has often been found strongly in Novæ. It occurs, in fact, somewhat regularly when hydrogen is present, and the conditions of excitation may be expected to be strong either in the electrical or thermal sense.

Perhaps it has not been isolated directly in the laboratory. Wright is of opinion that Dufour‖ observed it, but assigned it wrongly to the secondary spectrum of hydrogen, which, of course, does not accompany it in celestial spectra. At least, Dufour registered a spectrum of a continuous type in the ultra-violet with an origin certainly hydrogen in some form.

The suggestions we wish to advance with regard to its further experimental examination are not, perhaps fortunately, dependent on its production in the laboratory. This is due mainly to the success which has been obtained by Mr. H. H. Plaskett¶ in the adaptation of the wedge method** of spectrophotometry to the special problems presented by celestial spectra. The fundamental need is a determination of the law of intensity distribution along the spectrum, according to an absolute scale which can be founded on the radiation from such a source as the carbon-arc.†† The bearing of such a determination will appear later. The production of the spectrum in a pure manner in the laboratory for the purpose of such a determination presents great difficulty, in view of the spectrum of water-vapour which extends beyond the Balmer limit, and which would overlap in critical regions. Moreover, the secondary hydrogen spectrum provides a great complication on the red side of the Balmer limit, where a very special accuracy is desirable.

* *Proc. Amer. Phil. Soc.*, 49, 530, 1920.
† *Phil. Trans.*, A, 197, 389, 399.
‡ *Ap. Journ.*, 29, 100, 1909.
§ *L.O.B.*, 9, 54, 1917.
‖ *Ann. Chem. et Phys.*, 9, 361, 1906.
¶ *Pub. Dom. Astrophys. Obs.*, 2, No. 12.
** Merton and Nicholson, *Phil. Trans.*, A, 216, 459 (1916).
†† *Phil. Trans.*, A, 217, 237 (1918).

The absence of these two spectra from the celestial spectra which show the continuous hydrogen spectrum shows effectively that they furnish the first point of attack, after the manner of Plaskett, rather than a similar spectrum produced in the laboratory, where these disturbing agencies cannot be eliminated.

In the publications of the Lick Observatory, many data may be found regarding the astronomical occurrence of this spectrum and its general limits in the scale of wave-length. Its red limit is always on the red side of λ3646, as the following observations, quoted from Wright's paper as typical, will serve to show :—

Evershed records the limit in the chromosphere at λ3668, about 22 Å. towards the red, and 7 Å. beyond the most ultra-violet line he recorded.

In the nebula N.G.C. 7009 it is at λ3650, and in others, though not precisely estimated, it is yet more towards the red.

As an absorption spectrum it begins in α Cygni at λ3710 and has a maximum at about λ3660, and in Vega it begins at λ3800 and has its maximum at λ3710.

Wright notes that the spectrum always starts very near to the position of the last recorded line of the Balmer series, and, adopting the hypothesis after Bohr, that there is a limit to the number of Balmer lines which can exist, owing to the finite nature of the mean free path of an atom in an assemblage, obtains a reason for this effect.

Bohr's conclusion that the number of lines of the Balmer series available for observation is determined by the density of the gas—and the consequent explanation of the number seen, for example, by Dyson in the solar chromosphere—is not entirely satisfactory, in view of the experiment described by Merton and Nicholson,* who observed 14 sharp lines of the series in a tube filled mainly with helium at the remarkable pressure of 42 mm. of mercury. Calculation shows that under these conditions the radius of a hydrogen atom emitting the last member, on the principles of Bohr's theory, is twice the average distance apart of the atoms. This paradox has never been explained, but it is sufficient to show that the density is not the ultimate agency which determines the number of lines which can be seen. R. W. Wood has extended the hydrogen series even further in the laboratory, under conditions which preclude any correspondence between the number of visible lines and the density of the gas in the tube. The agency which is in fact effective in limiting their number must be of a different nature, for it is clear from experiments of this type that there is a feature present in a "bound" electron of an atom which assigns it and its behaviour definitely to that atom, even when another atom is so close as to be effectively intertwined with the first.

The nature of the agency, however, which limits the number of lines obtained is perhaps not very relevant. We merely assume that an atom can reach a certain radius—proportional to the square of its quantum number—and no more, while the extreme line shown arises from a fall inwards of an electron from this radius. It is natural to go further and

* *Proc. Roy. Soc.*, A, 96, 116 (1919–20).

suppose that an electron cannot become "bound" to the atom until it approaches within a distance equal to this radius. After this manner, it will be found that Wright's theoretical argument, which we shall now discuss briefly, is not in fact dependent on the hypothesis of density.

The fundamental basis of the theory is that the spectrum arises from the actual "binding" of stray electrons by atoms, and the consequent emission of energy in the process. An electron may, before "binding," have any velocity, and therefore kinetic energy, in excess of the amount required for its motion in the path in the atom to which it is bound. This must be emitted with an appropriate frequency, and it can have a continuous set of values, so that the frequencies form a continuous set, with no probable gaps, and therefore a continuous spectrum.

The frequency emitted in the binding of a given electron is proportional to the amount of energy it must spare. The greater the energy, the greater the frequency, and the consequent shift of the radiation towards the ultra-violet. Dr. Wright calculates the energy required to develop a frequency corresponding to the limit λ3340 of planetary nebulæ. Crediting the electron with the "average energy" corresponding to a given temperature, he finds that the necessary temperature of the gaseous material in the nebulæ is about 2500° C. This corresponds to the electrons moving most rapidly before capture, and yet sufficient in number to develop a visible radiation on a plate. A consideration of the limit at the red end and the general distribution of the spectrum indicate that a temperature of 1000° C. would be sufficient to produce the spectrum as ordinarily observed. This is by no means an unlikely temperature in the material of planetary nebulæ.

A quotation of Bourget, Buisson, and Fabry's estimate * of the temperature of the Orion nebula, found by the method of limiting interference of the emitted lines, requires, of course, to be received with caution in view of our present knowledge of spark spectra. It was founded on the supposition that the lines employed were subject only to the kinetic theory type of broadening calculated by the late Lord Rayleigh and by Schönrock. This implies that the shape of an intensity curve of a line, near its maximum $\lambda = \lambda_0$, is given by

$$I = Ae^{-\kappa(\lambda-\lambda_0)^2},$$

where I is the intensity at wave-length λ. Under spark conditions, Merton and Nicholson † showed that the Stark effect, even though not visually present and evidenced by clear separation of components, becomes the determining feature in the shape of this curve and follows the quite different law for each component,

$$I = e^{-\kappa(\lambda-\lambda_0)},$$

which vitiates calculations of temperature of the material emitting spark radiation—the type now generally believed to occur in these nebulæ.

But, at the same time, the supposition of a possible temperature of

* *Ap. Journ.*, **40**, 258 (1914).
† *Phil. Trans.*, A, **217**, 237 (1918).

1000° C. would be, at the present day, regarded generally as reasonable, so that there is at least an *a priori* case for the suggested explanation of this peculiar spectrum.

The law of intensity across this continuous spectrum is perhaps one of the most fundamental problems for investigation. Certainly no other can throw greater light on the unsolved problems of the quantum theory, with regard to which we can at present only make assumptions, without theoretical basis, so long as they meet the experimental evidence. This spectrum belonging to hydrogen, on which, in its atomic state, the quantum theory is alone quite precise, affords a field more directly valuable than even the secondary spectrum, involving, as the latter does, the unknown structure of the molecule. If the law of distribution of velocities among electrons liable to capture is that of the kinetic theory, as would be expected, and if all electrons whatever their velocity were equally liable to capture, the law of distribution of intensity is known theoretically, and its verification would solve the second main problem and give much insight into the ultimate mechanism. There are already many indications that in the capture of an electron its previous history is of no account. But a fundamental problem of a converse nature is raised here. If an electron with any velocity is equally liable to capture, it may be able to leave an atom with any assigned initial velocity—a conclusion at variance with experimental facts. We have shown elsewhere* that an electron cannot leave an atom with a quantised path in any hyperbolic (or parabolic) orbit about the atomic nucleus, and that consequently it must be regarded as "unbound" from the moment at which it leaves its stationary orbit. The unbinding thus takes place in such a way as to give it no change of magnitude in velocity, but only change of direction, in the first instance. The relative liability to capture, as a function of initial velocity, could probably be found from the study of an intensity curve in the continuous hydrogen spectrum.

Only one further point needs to be considered briefly. The extreme red side of the spectrum, usually sharp, is, on Wright's view, behind the limit of the Balmer series because due to captured electrons whose original velocity was zero. If the n^{th} observed line indicates the limiting radius of the atom, already discussed, an electron reaching this distance with no kinetic energy can only be captured by falling into one of the next inner orbits. It is in defect, by the kinetic energy of orbital motion in the n^{th} orbit, as regards an electron already bound in that orbit. The frequency of radiation emitted by its fall is, therefore, smaller than that of the corresponding series line. It follows that the end of the continuous spectrum should be on the red side of the last series line observed. This is an effective explanation of the fact that the series and continuous spectra do overlap. The amount of overlapping can be calculated at once, and should be equal to the interval, in wave number, between $\lambda 3646$ and the last observed series line. This is very closely the case.

This paper will have served its main purpose if it calls the attention of astronomers to an opportunity to contribute very much to atomic physics.

* *Phil. Mag.*, 1922 July, p. 193 *et seq.*

The Proper Motion of the Cluster N.G.C. 2168 (*M* 35).

By W. M. Smart, M.A., D.Sc.

(*Communicated by the Director, Cambridge Observatory.*)

§ 1. The cluster N.G.C. 2168 (M 35) belongs to Group II. in the classification of P. J. Melotte (*Memoirs R.A.S.*, **60**, 180). Melotte describes the general appearance of clusters of this class as follows: "Loose clusters, having a well-defined outline." As regards N.G.C. 2168 in particular, his description is: "A large open cluster, well defined and condensed toward the centre." Inspection of the "Carte photographique du Ciel," Zone +24°, No. 46 (Paris), shows, however, that within 2′ or 3′ of the centre (R.A. $6^h\ 2^m\cdot7$; Dec. +24° 21′) the number of stars is much smaller than one would expect if the cluster were actually condensed *at* the centre. This feature has been pointed out by Sigfrid Raab in his "Research on Open Clusters" (*Medd. fran Lunds Astr. Obs.*, serie ii., No. 28, p. 29). Raab's description is as follows: "A splendid specimen. A very large, thin and circular cluster, without sharply conspicuous condensation. On the contrary, the centre seems to be rather void of stars. In the vicinity of the centre is a gathering of 5 or 6 stars which are not to be clearly distinguished. The cluster passes imperceptibly into the environs." The diameter, according to Melotte, is 40′. Raab gives a somewhat higher value. In the present investigation I have adopted as diameter the value 45′.

§ 2. The parallax of the cluster has been determined by Raab (*loc. cit.*) and H. Shapley (*Communications to the Nat. Acad. of Sciences*, No. 62). Their results are 0″·0019 and 0″·0006 respectively. Raab bases his investigation for N.G.C. 2168 on the A-type stars. From the number of the brighter A stars in the cluster recorded in the *Henry Draper Catalogue* and from the total counts of stars (according to magnitude) in the cluster, the numbers of A-type stars for magnitudes 10·5 and 11·5 are inferred from rather general considerations. The application of a dispersion formula leads to the value of the parallax given above. Shapley's method, on the other hand, depends on the observed colours and spectra of the stars in the cluster, and the estimated absolute magnitudes are then directly correlated with the apparent magnitudes. A parallax of the order of 0″·001 is indicated by both investigators, and a small but possibly definite proper motion might be expected, with a fair amount of confidence, to reveal itself in plates separated by an interval of 20 years or so.

§ 3. Several excellent plates of the cluster and the surrounding region had been taken over 20 years ago by A. R. Hinks, and these were combined with recent photographs (taken through the glass) in the following manner:—

	Plate No.	Date.
I.	154	1903 March 5
	1106	1921 ,, 2
II.	151	1903 February 18
	1111	1921 March 10
III.	151	1903 February 18
	1504	1924 January 27

The earlier plates had an exposure averaging 20 minutes and the recent plates 1 hour. In these plates the cluster is in the centre. The measurements throughout were made independently by Mr. H. E. Green and the writer. Forty stars outside the limits of the cluster were selected for the determination of the plate constants. The reductions were according to the method followed in my previous investigations on Proper Motions. The diameters of the stellar images were measured to assist in the identification of the stars with those appearing in the Paris *Catalogue Astrographique*, tome 1.

§ 4. The region lies partly in the Berlin section of the Astronomische Gesellschaft Catalogue (1875) and partly in the Cambridge section. There are 13 stars, common to the region and the *Greenwich Catalogue*, 1910; for these stars, proper motions have been determined principally by comparison with the A.G. positions and the Greenwich observations made, on the average, after an interval of 33 years.

If G_x and G_y denote the proper motions of one of these stars, in seconds of arc per century, parallel to the equator and in declination respectively, and if M_x and M_y are the corresponding measures of relative proper motion derived from the comparison of the plates, then each of the 13 stars gives a pair of equations

$$\xi = M_x - G_x$$
$$\eta = M_y - G_y,$$

where $(-\xi, -\eta)$ are the quantities—in the mean—to be applied to the components of the relative proper motions to convert them into absolute proper motions.

The following Table I. shows the comparison.

TABLE I.

Comparison of the Relative Proper Motions with the Greenwich Proper Motions.

B.D.	Greenwich No.	Mag.	M_x.	G_x.	M_y.	G_y.	ξ.	η.
24°			″	″	″	″	″	″
1092	2839	8·7	−0·2	+2·6	−0·2	−0·7	−2·8	+0·5
1095	2840	8·6	+1·0	+1·8	+1·4	+0·6	−0·8	+0·8
1096	2842	9·0	+0·5	+0·9	+1·0	−0·9	−0·4	+1·9
1109	2860	8·7	+0·2	+2·7	−0·4	+0·6	−2·5	−1·0
1123	2874	8·6	−0·2	−0·3	+1·1	−2·3	+0·1	+3·4
1126	2878	8·2	−0·4	+5·0	+1·0	−2·2	−5·4	+3·2
1128	2880	8·6	−1·4	−1·2	+0·4	−2·3	−0·2	+2·7
1138	2888	9·4	−0·9	−3·1	+1·1	+1·5	+2·2	−0·4
1148	2904	8·2	0·0	+1·1	0·0	−0·5	−1·1	+0·5
1151	2912	5·9	−0·7	+0·7	−3·8	−5·6	−1·4	+1·8
1153	2921	8·7	−0·4	+1·5	+0·8	−1·9	−1·9	+2·7
1156	2926	9·2	−0·3	+4·8	+1·6	−2·0	−5·1	+3·6
1159	2931	9·0	−1·8	+2·2	+0·4	−1·5	−4·0	+1·9

The mean values of ξ and η derived from Table I. are given by

$$\xi = -1''\cdot8 \pm 0''\cdot4$$

and

$$\eta = +1''\cdot7 \pm 0''\cdot3,$$

the probable errors of a single determination of ξ and η being $\pm 1''\cdot5$ and $\pm 1''\cdot0$ respectively.

The values of η in the last column of Table I. are in satisfactory agreement, and the mean value of η, just derived, is substantially the same as that obtained from the investigation of the parallactic motions (see § 5). On the other hand, the values of ξ show a much larger dispersion about the mean, and the mean value of ξ obtained above is markedly different from the value deduced from the parallactic motions. It will be shown subsequently that the former value of ξ (that derived from Table I. above) must be regarded as less reliable than the latter. It is of interest to note that the probable errors of ξ and η are almost precisely the same as those found in a similar investigation of proper motions in the region of B.D. 30°3639 (*M.N.*, **84**, 129).

§ 5. The parallactic motions P_x, P_y—in seconds of arc per century—of a star of visual magnitude m (Harvard scale) are derived from the formulæ

$$P_x = 4\cdot10\pi_m \sin\lambda \sin\chi,$$
$$P_y = 4\cdot10\pi_m \sin\lambda \cos\chi,$$

in which π_m is the parallax of the star and λ and χ are respectively the angular distance and position-angle of the star with reference to the ant-apex of the solar motion. For the region under consideration,

$$\lambda = 59^\circ$$
$$\chi = 181^\circ$$

and the galactic latitude $= +2^\circ$.

The values of π_m have been taken from *Groningen Publications*, **29**, table 26. Altogether there are 34 B.D. stars in the region outside the cluster area. These have been divided into 4 groups. If N is the number of stars in each group, the corresponding values of ξ and η are derived from the equations

$$\left.\begin{aligned} N\xi &= \Sigma M_x - NP_x \\ N\eta &= \Sigma M_y - NP_y \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (1)$$

in which ΣM_x and ΣM_y are the algebraical sums of the measured relative proper motions of the stars in each group. These equations imply, of course, a random distribution of the peculiar motions of the group stars.

The values of π_m, P_x, and P_y are shown below in Table II.

Table II.

Group.	Mean Mag. (H.S.)	Number of Stars N.	π_m.	P_x.	P_y.
I.	8·1	6	0″·0060	−0″·04	−2″·10
II.	9·0	5	0·0042	−0·03	−1·47
III.	9·5	10	0·0036	−0·02	−1·26
IV.	9·7	13	0·0034	−0·02	−1·19

In Tables III. and IV. below, the second columns give ΣM_x and ΣM_y for each of the magnitude groups. The fourth columns contain the values of $N\xi$ and $N\eta$ obtained from the formulæ (1) above. From all 34 stars the mean values of ξ and η are given by

$$\xi = +1''\cdot7/34 \quad \text{and} \quad \eta = +55''\cdot5/34$$

or

$$\xi = +0''\cdot05$$

and

$$\eta = +1''\cdot63.$$

TABLE III.

(*x Components.*)

N.	ΣM_x.	NP_x.	$N\xi$. Obsd.	$N\xi$. Cald.	Average Residual for each Star in Group.
	″	″	″	″	″
6	−1·4	−0·2	−1·2	+0·3	−0·2
5	+1·2	−0·1	+1·3	+0·2	+0·2
10	−4·4	−0·2	−4·2	+0·5	−0·5
13	+5·5	−0·3	+5·8	+0·7	+0·4
34	...	...	+1·7	+1·7	...

TABLE IV.

(*y Components.*)

N.	ΣM_y.	NP_y.	$N\eta$. Obsd.	$N\eta$. Cald.	Average Residual for each Star in Group.
	″	″	″	″	″
6	−1·4	−12·6	+11·2	+9·8	+0·2
5	−1·4	−7·4	+6·0	+8·2	−0·4
10	+4·7	−12·6	+17·3	+16·3	+0·1
13	+5·6	−15·4	+21·0	+21·2	0·0
34	...	...	+55·5	+55·5	...

The fifth columns of the preceding Tables III. and IV. are calculated from the derived values of ξ and η, viz. $+0''\cdot05$ and $+1''\cdot63$ respectively. The remaining columns are self-explanatory.

§ 6. Combining the results from §§ 4 and 5, we have the comparative Table V. giving the values of ξ and η from the two methods.

TABLE V.

	ξ.	η.
	″	″
From meridian proper motions of 13 stars	−1·8	+1·7
From the parallactic motions of 34 B.D. stars	0·0	+1·6

The values of η agree perfectly. The value $+1''\cdot6$ has been adopted.

When we consider the values of ξ as given by the two methods, we are confronted by a difficulty apparently hard, at first sight, to resolve.

Which is the more reliable method and which value of ξ is to be preferred? The two values cannot be reconciled, and the discrepancy is so large that one of the values must be rejected.

The proper motions given by M_x are relative to a satisfactorily large number of stars: we may describe these here as the reference stars. If we assume first of all that the distribution of the space motions of the reference stars is a random distribution, then the mean motion of the group should exhibit the solar motion reversed. In three previous papers (*M.N.*, 84, 3, 123, and 481) the evidence from the 11 photographic regions investigated seems unmistakable that the stars between magnitudes 8 and 12 exhibit the streaming features associated with the brighter stars. There is no *a priori* reason for suggesting that the stars in the neighbourhood of N.G.C. 2168 form an exception to what appears to be a fairly well-determined characteristic of the stars in general. Now the proper motions with which we are dealing here are comparatively small, so that the distributions of motions for the two hypotheses $\xi = 0''{\cdot}0$ and $\xi = -1''{\cdot}8$ should then be completely different. It is fortunate that the position of the region with regard to the apices of Drift I. and Drift II. makes for simplicity in the general argument. The position-angles of the apices are 181° and 191°; that is to say, the expected distribution will give sensibly a single-drift curve in position-angle 183° approximately, for it is scarcely to be anticipated that the extremely small separation between the directions of Apex I. and Apex II. will have any appreciable influence in changing noticeably the single-drift distribution. If we use $\xi = -1''{\cdot}8$, the distribution curve will have a pronounced and characteristic extension in position-angle 130° approximately, completely at variance with what one would naturally expect; whereas the value $\xi = 0''{\cdot}0$ leads undoubtedly to a single-drift curve in the direction anticipated. The latter curve is given in fig. 2.

In Table I. there are 4 stars—Greenwich 2839, 2860, 2904, 2931—which were observed at Cambridge by the late Mr. Graham for the A.G. Catalogue, and upon these observations the values of G_x tabulated principally depend. If a_G and a_C are the right ascensions (in arc) of a star—referred to the same epoch—as given by the Greenwich and Cambridge meridian observations respectively, the interval being 33 years, the value of G_x is given by

$$G_x = \frac{100}{33}(a_G - a_C)\cos 24^\circ$$

or

$$G_x = 2{\cdot}7\,(a_G - a_C).$$

Considering only the 4 stars mentioned, we find, for the mean, that $\xi = -2''{\cdot}6$; consequently

$$\xi \equiv M_x - 2{\cdot}7a_G + 2{\cdot}7a_C = -2''{\cdot}6 \quad . \quad . \quad . \quad (2)$$

It is certain that the values of a_G are of a very much higher order of accuracy than the values a_C, for the observing methods as well as the number of observations were greatly to the advantage of the former;

therefore it may be assumed that the values of a_G, as compared with the corresponding values of a_C, are sensibly correct. The internal evidence from the measures of the star images rules out of court errors of the order of magnitude in M_x required by equation (2). It may be presumed, then, that the principal source of error is in the values of a_C. It is well known, from the extensive discussions of Professor Turner and Mr. Hinks, that Mr. Graham's observations reveal a considerable magnitude equation for the fainter stars of the catalogue. For the 4 stars with which we are at the moment concerned—of average magnitude 8·7—the correction to be applied to the observed right ascensions, on the score of magnitude equation, would be about $-0^s{\cdot}08$, and this correction (expressed in arc) is supposed to be incorporated in the values of a_C given in equation (2).

If A_C is a hypothetical value of a_C such that the value $\xi = 0''{\cdot}0$ is given by (2), then we have

$$\xi \equiv M_x - 2{\cdot}7a_G + 2{\cdot}7A_C = 0''{\cdot}0,$$

so that

$$2{\cdot}7(A_C - a_C) = +2''{\cdot}6,$$

from which we have (reverting to time-measure)

$$A_C - a_C = +0^s{\cdot}06.$$

This last result would seem to show that the magnitude equation, so far as this small numerical sample of the Cambridge stars is concerned, has been overestimated by about $0^s{\cdot}06$, and should, consequently, be actually of the order of $-0^s{\cdot}02$ approximately for the stars considered.

The remaining 9 stars of Table I.—in the Berlin zone—show an average discordance of $+0^s{\cdot}03$ approximately (represented as residual magnitude equation) on similar assumptions to those made in connection with the Cambridge stars.

It is not suggested that the magnitude equation based on the observations of a very large number of Cambridge or Berlin stars does not, in the mean, fulfil the requirements demanded by later and more accurate determinations of position, but rather that the mean corrections as applied to a small group of stars may be considerably erroneous and so lead to the inconsistencies, revealed by the stars of Table I., in the values of ξ derived by two wholly independent methods.

The application, in practice, of this brief discussion, based on a limited quantity of material, is that only the most reliable proper motions should be utilised for the process of converting relative proper motions into absolute proper motions.

§ 7. As regards the present investigation, the arguments of the preceding section conclusively show that the change of origin (that is, in effect, the values of ξ and η) deduced from the parallactic method is much more in accordance with expectation and probability than the result derived from the meridian proper motions. Consequently, I adopt the value $\xi = 0''{\cdot}0$; that is to say, the relative proper motions parallel to the equator are also, for this region of the sky, actually the absolute proper motions. In declination, the relative proper motions

are converted into absolute proper motions by applying $-1''{\cdot}6$ per century to the former.

§ 8. Within the assumed limits of the cluster (diameter = 45′) there are 200 stars for which measures have been made on at least one pair of plates. Some of these must be field stars, unconnected, except in projection, with the cluster. The proper motions afford the only available material for effecting the separation of field stars from the cluster

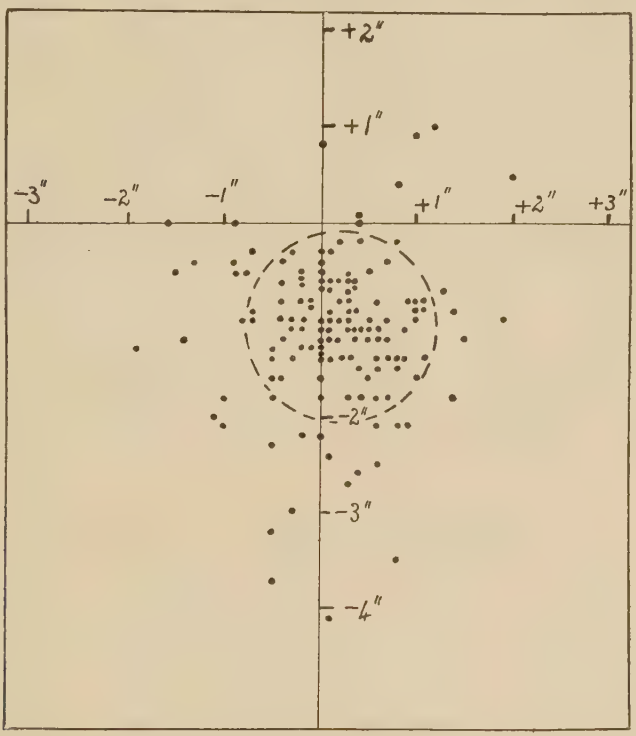


Fig. 1.—The Proper Motions of Stars within the Cluster-area.

stars. The latter should show a common motion, whereas the former should give a scattered distribution.

In order to determine the proper motion of the cluster, or, what amounts to the same thing, to derive from the data a reliable criterion whereby the two classes of stars within the cluster area may be separated, I considered it advisable to reject from the 200 stars, referred to above, all those which had not been measured on all three pairs of plates. The number remaining is 144. The proper motions of these stars are shown in fig. 1.

It will be immediately noticed that there is an unmistakable condensation of proper motions about the values $\mu_x = +0''{\cdot}2$, $\mu_y = -1''{\cdot}0$. This point was at first estimated by eye and a circle drawn about the

point as centre with diameter 1″·3. As the probable errors of the measures are about ±0″·6 in each co-ordinate, this circle should indicate fairly accurately — at least as a first approximation — the separation of the cluster stars from the field stars. The mean proper motion of the stars with proper motions lying within the circle was then found, and a circle with this latter point as centre and diameter 1″·0 was finally drawn, as in fig. 1. It is then assumed that all the stars within the circle belong to the cluster. This is a somewhat stringent condition, and it is possible, therefore, that a few cluster stars have been excluded. The conclusion from the counts is that, of the 144 stars considered, 100 belong to the cluster, the remaining 44 being field stars. This latter number gives about 100 stars per square degree, which is in good agreement with the average number (excluding the cluster stars) per square degree taken over the whole plate; if the effective area of the plate is assumed to be $1\frac{1}{4} \times 1\frac{1}{4}$ square degrees, the proportion actually found is 90 per square degree. It is possible too, having regard to the direction of the ant-apex, that a few stars have been erroneously included as members of the cluster, but the observational data in this investigation are insufficient to lead to definiteness on this point.

An interesting point is to investigate whether there is any evidence of a magnitude effect revealed by the 100 stars. Accordingly, the stars were divided into 7 magnitude groups. Table VI. shows the results. The last two columns give $\bar{\mu}_x$ and $\bar{\mu}_y$—the mean proper motions for each of the groups.

TABLE VI.

Group.	Mean Photographic Mag. (Paris).	No. of Stars.	$\bar{\mu}_x$.	$\bar{\mu}_y$.
m m	m		″	″
8·0– 8·4	8·0	2	+0·1	−0·4
8·5– 8·9	8·7	5	+0·1	−0·8
9·0– 9·4	9·3	6	+0·1	−1·0
9·5– 9·9	9·7	23	+0·3	−1·0
10·0–10·4	10·2	14	+0·1	−0·9
10·5–10·9	10·7	28	+0·2	−1·1
11·0–11·4	11·2	22	+0·2	−1·0

There is evidently no sign of a magnitude equation in either co-ordinate.

If X and Y denote the proper motion of the cluster, as a whole, in seconds of arc per century, parallel to the equator and in declination, the combined data of Table VI. give the proper motion of the cluster, as follows:—

$$X = +0''{\cdot}2 \text{ per century.}$$
$$Y = -1''{\cdot}0 \quad ,,$$

§ 9. The distribution of proper motions of the non-cluster stars in the 10° sectors $\theta - 5°$ to $\theta + 5°$, etc., is given in the columns headed (a)

in the following table. This is based on the adopted values $\xi = 0''{\cdot}0$ and $\eta = +1''{\cdot}6$. The full-line curve in fig. 2 is derived by taking means of three consecutive sectors. As has been remarked previously, this curve is virtually a single-drift curve in the direction anticipated.

TABLE VII.

Distribution of Proper Motions of Non-Cluster Stars.

θ.	Number of Stars. (a).	(b).	θ.	Number of Stars. (a).	(b).
10°	0	2	190°	13	7
20	0	7	200	8	4
30	0	2	210	7	2
40	0	3	220	6	4
50	2	4	230	5	6
60	1	3	240	8	5
70	2	4	250	4	4
80	3	4	260	1	4
90	4	4	270	2	5
100	3	5	280	0	1
110	3	3	290	1	4
120	7	3	300	1	5
130	7	3	310	0	3
140	7	2	320	0	4
150	4	4	330	0	1
160	10	5	340	1	1
170	15	12	350	0	2
180	14	12	360	1	3

§ 10. It remains to consider whether the proper motion of the cluster derived in § 8 is a real proper motion or not. The greater part of the proper motion is in declination, and it will be sufficient to consider this component. Clearly, the question depends, to a very large extent, on the value of η. If we had found that $\eta = +0''{\cdot}6$, the proper motion of the cluster would have virtually disappeared. But it is significant that precisely identical values of η have been derived by two independent methods; and, moreover, the adopted value of η, viz. $+1''{\cdot}6$, has led to a distribution of proper motions of the non-cluster stars similar, in character, to the distributions observed in other regions of the sky. As a further test we can reverse the process. Let us assume that the proper motion of the cluster is zero, and then investigate the distribution of proper motions of the non-cluster stars. This is equivalent to putting the origin according to the values $\xi = +0''{\cdot}2$, $\eta = +0''{\cdot}6$. The number

of stars in each of the 10° sectors is given, on this hypothesis, in column (*b*) of Table VII. If a single-drift curve is the result, then the reality of the proper motion of the cluster is open to doubt. The broken curve in fig. 2 gives the distribution of proper motions on this hypothesis, and, as might be expected, it indicates a limited or residual

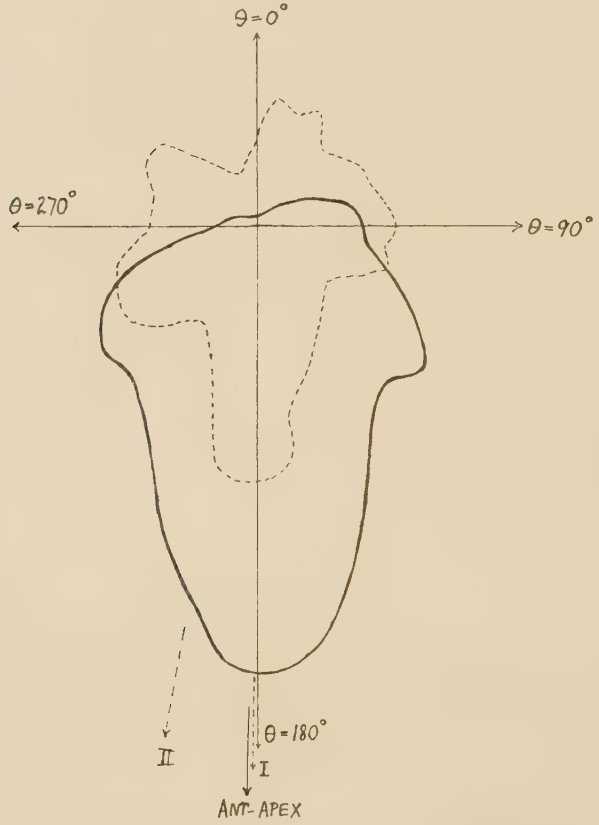


FIG. 2.—The Distribution of Proper Motions of the Non-cluster Stars.

Full curve: according to column (*a*), Table VII.
Broken „ „ „ (*b*) „

streaming in position-angle 180°. This is due to a small excess of stars in this direction, as shown at once in Table VII. From the proper-motion plot it is clear that, as the value of η is reduced, this partial extension of the curve in direction 180° becomes less and less prominent. Fig. 2 shows conclusively that whereas the value $\eta = +1''{\cdot}6$ leads to a purely single-drift curve, as the circumstances of the region would suggest, the value $\eta = +0''{\cdot}6$ gives a distribution characteristically different from all previous results.

The conclusion seems definite that the proper motion of the cluster is real, and is of the order given in § 8.

§ 11. The following table gives the numbers of non-cluster stars with total absolute proper motions between 0″·0 and 1″·0, etc., per century.

TABLE VIII.

Total Proper Motion.	No. of Stars.
″ ″ 0·0–1·0	29
1·0–2·0	59
2·0–3·0	34
3·0–4·0	11
4·0–5·0	4
5·0–6·0	3

Observatory, Cambridge:
1925 *January* 1.

A New Method for determining the Positions and Proper Motions from a Great Number of Catalogues. By Dr. J. E. Baron de Vos van Steenwijk.

The determination of positions and proper motions from a great number of catalogues by a least-square solution involves a considerable amount of labour, so that each way of avoiding it, even at a small loss of accuracy in the results, is well worth examining. It is a question of efficiency whether the saving of time and expense balances the loss of accuracy. I think the method I suggest meets this requirement.

The simplest way is to plot all catalogue places for each co-ordinate against time and to draw a straight line as nearly as possible through all points. This is done most easily by moving a stretched thread across the paper till it occupies the required position and reading off two ordinates, say 100 years apart. This is being done at Bergedorf for the stars of the *Geschichte des Fixsternhimmels.* But it has amongst others the serious drawback that no account is taken of the different weights of the catalogue places. To meet this objection I have modified it in the following way.

Instead of representing each place by a point, I draw a vertical line [time being represented on the axis of abscissæ] on each side of the point, giving it a length proportional to its probable error. In stretching the thread one must now endeavour to make $[p\Delta\Delta]$ a minimum, where p represents the weight. But, the lengths of the lines being inversely proportional to the square root of p, this means that *if we measure the deviation* Δ *(of each point from the thread) with the length of its vertical line as a unit, those deviations squared must give a minimum.*

It is a question of experiment whether the eye can be trained to

judge at once in what position of the thread this condition is fulfilled. A test I made on 43 stars selected at random from my catalogue of 2840 southern stars (*Ergänzungshefte zu den Astronomischen Nachrichten*, Band 4, No. 7), convinced me that this can be acquired by practice and that the method then yields very useful results.

Those stars appear each in 5 to 10 catalogues from 1823 to 1920, and the weights cover a range from 0·05 to 3·00,* which makes it a severe test for my method.

The position of the thread was read off at three points, viz. 1820, 1875, and 1920. The difference of the first and the last gives the centennial proper motion, the second reading served as a check, and, as all declinations were reduced to the equinox of 1875, it gives the mean position at that date.

Afterwards a rigorous least-square solution was made for δ and μ_δ, and the results are compared below. For δ only the differences are given.

It was, however, pointed out to me that proper motions would not satisfy astronomers if they did not get probable errors as well. The problem was now to get these as easily as possible. Using the ordinary symbolism, and putting $\delta = x$ and $100\,\mu_\delta = y$, the probable error on the scale of the P.G.C. is $0''{\cdot}30\,\sqrt{[\beta\beta]}$ if $[\beta\beta] = \frac{1}{[bb] + a_b[ab]}$, where $a_b = -\frac{[ab]}{[aa]}$. Now $[bb]$ can be calculated very easily, if we simply suppose that all observations of a single catalogue have been made at the same epoch. The same factors then occur continually and can be tabulated. (Of course this supposition is *not* made in the construction of the graph.)

If now the (weighted) mean epoch for each star was known beforehand and could have been chosen as date of comparison, $[ab]$ would be zero. This of course is impossible, but the second term of the denominator can be kept reasonably small by choosing 1900 as date of comparison. The mean epoch varies between 1893 and 1905, and $a_b[ab]$ never exceeds 10 per cent. of $[bb]$ (always subtractive). This correction can be estimated at a glance from the catalogues represented in each case without any further calculation. The probable error can then be tabulated with $[bb1]$ † as an argument, and so the labour is minimised in a very effective manner. The results of this approximation are compared with the probable error from the least-square solution in our table, and show how well this method works.

We must, however, understand that these probable errors cannot be applied directly to our graphical proper motions, as they depend on the least-square solution, which is not strictly carried out. As the graphical p.m. show a *mean* deviation of $0''{\cdot}38$ from the calculated values, this can be assumed to give an additional *probable* error of $0''{\cdot}32$. If we do this, the mean value of the p.e. for $100\,\mu_\delta$ increases from $0''{\cdot}62$ to $0''{\cdot}70$, which means a loss of weight in the proportion of 5 to 4.

I do not think this loss of weight is to be deemed excessive if we consider the enormous saving of labour: no formation and solution of

* On the scale of the Prel. Gen. Cat.

† $[bb1] = [bb] + a_b\,[ab]$.

normal equations, but only a graph, which it takes a few minutes to draw and read off. [If the drawing is made on transparent paper, the same piece of squared cardboard can be used over and again.]

Another test to which I have subjected my measures makes it probable that the results are in reality still better. The last columns of the table contain the sum of the squares $[p\Delta\Delta]$ of the residuals by each method. The excess of this quantity for the graphical method is much less than one would expect, and points to a loss of weight in the proportion of 11 to 10. [The small negative differences for some stars cannot, of course, be real, and are occasioned by the rounding off to tenths of a second.]

I think this can be explained by the following considerations. If the δ and μ_δ from a least-square solution are substituted in the equations of condition, we get a system of residuals Δ that has a maximum probability. But if we examine the change of probability due to a slight change in δ and μ_δ, we see that the effect is very different, even in cases where we have the same probable error. In other words, the maximum of probability, or the minimum of $[p\Delta\Delta]$, may be very steep or very flat. In the latter case a small change of δ and μ_δ does not really matter much, and those are the cases when it is most difficult to decide which position of the thread is the best.

So I conclude that the graphical method is at its best in cases where a slight error harms most, and this certainly is a great point in its favour.

I want to emphasise that this behaviour of $[p\Delta\Delta]$ does not depend on the absolute value of its minimum, and is therefore entirely overlooked in a least-square solution. But it shows at once in a graph, and it seems to me that it ought to influence our appreciation of the resulting proper motion

In the following table Comp. or C. means computed, and Graph or Gr. means obtained graphically as indicated in this paper.

Cat. No.	$\Delta\delta$. Comp.–Graph.	100 μ_δ. Comp.	100 μ_δ. Graph.	Diff. C.–Gr.	p.e. Comp.	p.e. Appr.	$[p\Delta\Delta]$. Comp.	$[p\Delta\Delta]$. Graph.
	″	″	″	″	″	″	″	″
1	+0·2	−14·7	−13·7	−1·0	0·55	0·56	1·22	1·63
2	−0·1	+ 5·6	+ 5·4	+0·2	0·83	0·80	1·57	1·58
3	+0·1	− 2·1	− 1·4	−0·7	0·54	0·54	2·23	2·27
4	−0·2	+ 0·5	− 0·7	+1·2	0·79	0·78	1·90	2·15
5	−0·2	− 9·4	− 9·5	+0·1	0·62	0·62	1·38	1·76
6	+0·1	− 0·9	− 0·9	0·0	0·45	0·46	0·98	1·00
7	−0·1	+ 3·2	+ 3·2	0·0	0·67	0·66	0·48	0·46
10	+0·1	+ 0·2	+ 0·8	−0·6	0·70	0·68	1·05	1·15
11	0·0	+ 3·9	+ 4·2	−0·3	0·46	0·46	0·14	0·37
12	0·0	+ 1·8	+ 1·8	0·0	0·60	0·61	2·06	2·06
13	+0·1	− 2·3	− 1·7	−0·6	0·84	0·81	0·36	0·39

Cat. No.	$\Delta\delta$, Comp.–Graph.	100 μ_δ. Comp.	100 μ_δ. Graph.	Diff. C.–Gr.	p.e. Comp.	p.e. Appr.	$[p\Delta\Delta]$. Comp.	$[p\Delta\Delta]$. Graph.
	″	″	″	″	″	″	″	″
15	+0·2	− 4·8	− 4·0	−0·8	0·54	0·52	2·38	2·66
16	+0·2	−14·4	−14·7	+0·3	0·56	0·57	0·90	1·24
18	−0·2	− 2·4	− 2·9	+0·5	0·67	0·65	0·96	0·97
19	+0·1	+ 5·8	+ 6·3	−0·5	0·55	0·54	0·99	1·07
20	−0·2	− 3·9	− 4·6	+0·7	0·49	0·51	1·69	1·82
21	−0·3	+ 0·3	− 0·5	+0·8	0·64	0·65	1·31	1·81
24	−0·2	+ 1·0	+ 0·9	+0·1	0·58	0·61	2·67	2·66
25	0·0	− 0·3	− 0·6	+0·3	0·67	0·60	0·24	0·26
26	−0·1	+ 2·2	+ 2·1	+0·1	0·61	0·63	0·36	0·30
28	+0·1	− 0·9	+ 1·5	−0·6	0·59	0·57	0·45	0·58
29	0·0	+ 1·2	+ 1·4	−0·2	0·47	0·48	1·24	1·19
30	0·0	− 4·6	− 5·1	+0·5	0·61	0·62	0·88	1·00
38	0·0	− 0·9	− 0·8	−0·1	0·55	0·56	0·52	0·58
39	−0·2	− 1·3	− 1·6	+0·3	0·52	0·51	1·46	1·55
46	0·0	− 6·1	− 6·3	+0·2	0·56	0·55	0·21	0·25
47	−0·2	− 2·4	− 2·7	+0·3	0·52	0·51	1·76	1·77
48	0·0	+ 2·2	+ 2·2	0·0	0·52	0·54	1·26	1·26
52	−0·3	− 1·8	− 3·0	+1·2	0·56	0·55	2·84	3·16
56	0·0	− 0·8	− 0·8	0·0	0·76	0·78	0·51	0·51
57	0·0	+ 1·7	+ 1·3	+0·4	0·84	0·81	1·46	1·53
59	−0·2	−10·3	−11·0	+0·7	0·44	0·44	1·70	2·33
60	0·0	+ 2·6	+ 2·3	+0·3	0·57	0·56	3·06	3·04
64	0·0	+ 0·6	+ 0·6	0·0	0·78	0·77	0·36	0·38
66	−0·1	+ 4·3	+ 4·0	+0·3	0·57	0·58	1·36	1·44
67	0·0	− 8·3	− 8·1	−0·2	0·67	0·66	0·42	0·43
67A	0·0	− 4·8	− 4·7	−0·1	0·81	0·81	1·44	1·46
74	+0·1	+ 4·2	+ 4·2	0·0	0·79	0·77	1·95	1·96
75	0·0	− 8·5	− 9·1	+0·6	0·66	0·64	0·25	0·36
76	0·0	+10·1	+10·5	−0·4	0·67	0·64	0·69	0·74
80	−0·1	3·4	− 3·9	+0·5	0·64	0·64	0·95	0·98
81	−0·1	+ 1·1	+ 1·0	+0·1	0·57	0·58	0·09	0·09
82	0·0	− 0·5	− 0·7	+0·2	0·61	0·63	1·17	1·20
			mean	±0·38	±0·62		Σ 50·90	55·40

Scheveningen:
1925 *January.*

Note on the Mean Longitude of the Moon. By J. Jackson, M.A., D.Sc.

(*Communicated by the Astronomer Royal.*)

In the *Monthly Notices* for 1924 November (p. 11) Mr. H. Spencer Jones has made a valuable discussion of the occultations observed at the Royal Observatory, Cape of Good Hope, 1880–1922. The observations were reduced first of all in reference to Hansen's Tables of the Moon, but the difference between Brown's and Hansen's Tables for each occultation was computed and applied. The residuals were analysed with respect to certain variables, and after the resulting terms were applied corrections to the mean longitudes of Brown's Tables for each year were obtained. The results are given in Table VI. (p. 34). By applying the mean difference for each year between Brown's longitudes and those of Hansen's Tables with Newcomb's corrections, the mean corrections to the latter tables were derived. The difference from the corrections derived from the Greenwich meridian circle and altazimuth observations given in the last column is in several years larger than might have been expected. For 6 of the 43 years the mean difference between the Greenwich and Cape results exceeds 1″. It is true that for most of these 6 years the Cape results have larger probable errors than in the other years, but still they are only ± ″·18, ″·27, ″·27, ″·30, ″·33, and ″·41. The Greenwich results depend on upwards of 100 transits a year, and, even allowing for a probable error of 1″·0 in each observation, the probable error of the yearly mean should not exceed 0″·10. In neither the Cape nor the Greenwich results does the yearly mean run so smoothly as it would be expected to do from its probable error, and apparently these differences are not explicable by the accidental errors of observation.

In the first place, consider the method of reduction in the two cases. That of the Cape observations has been explained above. The Greenwich observations were compared with Hansen's Tables with Newcomb's corrections, but afterwards periodic terms found by Dr. Cowell from a discussion of the Greenwich observations, 1851–1901 [see Greenwich volume for 1920, p. G iii], were applied. The periods of these terms are for the most part of the order of a month, except two of the order of a year. Their mean for a year should be small, and in any case they go some way towards reducing the short-period terms of Hansen to agreement with Brown.

Although the methods of reduction are so different it does not appear that the numbers in the final column of Jones' Table VI. would be seriously altered by a uniform system of reduction *if* the observations were uniformly spread over the year. From the run of the differences it appears that systematic differences in the star-places used might partly explain them, but as there are many reversals of sign, and both discussions were fundamentally based on Newcomb's equinox, this explanation is hardly tenable.

It appears that the only possible explanation is that short-period terms, together with non-uniform distribution of the observations throughout the year, particularly for occultations, make the yearly

mean unsatisfactory. It is difficult to believe that any short-period gravitational term with a large enough coefficient is missing from the theory, for, as has often been pointed out, it would require a large force to produce such a term. In view of the discussions to which the moon observations were subjected by Cowell, it is also difficult to think that such a periodic term exists in the observations.

The three most recent years for which the difference between the Cape and Greenwich results exceeds 1″ are 1904, 1909, and 1911. For the first two of these years there are respectively 34 and 41 occultations given in the Greenwich annual volumes. The equations of condition have been transformed into longitude and latitude, while the periodic corrections given by Cowell have been applied, the lunar parallax increased by 0″·40, and the adopted semidiameter by 0″·05 to agree with Jones' result.

The corrections to the longitude of the moon derived from the 41 stars observed in 1904 are as follows. In this calculation no observations have been rejected which were not rejected at the time of publication of the annual volume. Results from stars on the same night are bracketed, and those for which the coefficient of the longitude in the equation of condition was less than 0·5 are marked with an asterisk.

+8″·0	+6″·3	4″·1	6″·2	+4″·1
4·5	6·8	4·6*	3·8	3·5
0·8	6·5	1·8	1·1	1·9
4·8	4·8	3·6	4·2*	4·3
6·2	7·1		3·1	2·5
7·5	4·5		0·7	2·5*
7·2	6·7		4·8	+4·1*
4·0	7·4		4·8	−0·3
6·0	4·1		1·8	+4·5
			4·5	

The mean of all is +4″·4, as compared with +4″·07 from the transit circle and +5″·12 from the Cape occultations. The mean of the 18 up to July 9 is +5″·7, and of the 23 from August 3 is +3″·3. The mean by nights is +4″·8. The star which stands out most and gives the correction −0″·3 to the moon's longitude is Bradley 619, and five different observers got remarkably accordant observations of that occultation. It happens that the same star was occulted twice previously in 1904, and gave the corrections +6″·7 and +4″·5 to the moon's longitude, and on both these occasions the disappearance was at the bright limb, while the discordant observation was for a disappearance at the dark limb! On an examination of the reduction it was found that Newcomb's position of the star differed from Boss's position by a little over 2″, and that the correction to the moon's longitude when Boss's position is used is +1″·9 instead of −0″·3. If Boss's place of the star is used for the two occultations on the bright limb, the correction to the moon's longitude from them is increased to +7″·9 and

+5″·8. An analysis of the different nature of the phenomena reveals nothing particular. Disappearances on the bright limb give +4″·3, reappearances on the dark limb +4″·3, and reappearances on the bright limb +3″·8, the number of observations being 8, 8, and 2 respectively.

The results by nights for the occultations observed at Greenwich in 1909 are as follows, the numbers in brackets indicating the number of stars occulted. One star on the fourth last night was rejected. It was a reappearance at the bright limb observed with a rather small telescope, and gave a correction of −0″·9. It must have been late. The result bracketed is a poor observation of a reappearance.

+6″·7 (4)	[+ 1″·4 (1)]	+7″·4 (2)
+3″·0 (1)	+ 6″·9 (2)	+6″·8 (3)
+5″·4 (1)	+10″·6 (4)	+9″·0 (2)
+6″·5 (1)	+ 6″·8 (6)	+8″·2 (4)
+8″·6 (1)	+ 9″·9 (1)	

The mean by nights is +7″·4, by single stars +7″·5. The Cape occultations for this year gave +7″·87, the Greenwich transit circle and altazimuth +6″·53. The outstanding result in the Greenwich occultations is +10″·6 for 4 stars. These were all faint stars observed at reappearance on the dark limb, and were apparently early. The two rejected observations were reappearances at the bright limb; a single disappearance at the bright limb gave +5″·0, and a retained reappearance +7″·5. All the other phenomena took place at the dark limb, 18 disappearances giving a mean correction of +7″·3, and 12 reappearances of +8″·0.

This note further emphasises the fact that there are irregularities in the motion of the moon which are still unexplained. Frequently it may be possible to track down the major irregularities to errors in the positions of the stars occulted or to poor observations under difficult conditions. On the other hand, some occultations observed under very unfavourable conditions may be found quite satisfactory. The question of the rejection or retention of an observation is an extremely difficult one to settle. Reappearances of faint stars and phenomena of any kind at the bright limb are peculiarly liable to systematic error. The variation of the correction to the tables from year to year proves decisively that there are unexplained oscillations in the motion of the moon which may change the longitude of the moon by as much as 4″ in ten years, or 0″·4 a year. This will make the yearly mean unsatisfactory if the observations are not distributed fairly evenly throughout the year.

Occultations of Stars and Planets by the Moon, observed at the Royal Observatory, Greenwich, during the Years 1919–1924.

(*Communicated by the Astronomer Royal.*)

Date and G.M.T. of Observation.					Star and Phase of Occultation.		Mag. of Star.	Observer.	Aperture.
	d	h	m	s			m		in.
1919 Feb.	9	6	12	25·71	ι Tauri	D	4·7	A.C.	3·7
		6	12	25·14	,,	D	,,	H.A.	4·0
		8	34	15·56	105 Tauri	D	6·0	H.A.	4·0
	12	6	48	25·01	2B Cancri	D	6·0	A.C.	3·7
		8	38	29·15	5 Cancri	D	5·9	A.C.	3·7
1920 Sept.	18	7	49	44·89	123B Scorpii	D	6·5	A.C.	3·7
Nov.	21	8	48	6·45	Lalande 205	D	7·0	A.C.	3·7
	22	5	29	8·42	70 Piscium	D	7·8	A.C.	6·7
		5	41	36·72	ϵ Piscium	D	4·4	J.J.	10·0
		5	41	37·37	,,	D	,,	A.C.	6·7
		5	41	34·99	,,	D	,,	R.S.	6·5
1921 Jan.	15	7	57	23·44	116B Piscium	D	6·6	A.C.	3·7
Feb.	20	11	1	52·65	60 Cancri	D	5·7	R.C.	6·7
Mar.	18	10	55	8·96	67 Geminorum	D	6·7	A.C.	3·7
		10	55	8·62	,,	D	,,	R.C.	6·7
		10	54	59·13	68 Geminorum	D	5·2	E.M.	26·0
		10	54	59·13	,,	D	,,	R.S.	12·7
Aug.	8	8	7	59·64	*h* Virginis	D	5·4	A.C.	3·7
Oct.	6	6	52	17·47	Lalande 31462	D	6·6	A.C.	3·7
		6	52	17·03	,,	D	,,	R.C.	6·7
	16	10	56	43·02	B.D. +7°228	D†	9·3	C.D.	12·7
		11	5	33·57	B.D. +7°227	D†	9·1	C.D.	12·7
1922 Aug.	16	14	50	17·04	Aldebaran	D*	1·1	F.J.	6·7
		15	58	58·90	,,	R	,,	F.J.	6·7
Nov.	6	10	7	18·52	,,	D*	,,	A.C.	3·7
		11	19	33·46	,,	R	,,	A.C.	6·7
		11	19	32·64	,,	R	,,	A.C.	6·7
1923 Jan.	27	8	15	45·01	75 Tauri	D	5·2	H.A.	6·7
		8	30	42·62	θ^1 Tauri	D	4·2	A.C.	3·7
		8	30	42·12	,,	D	,,	J.J.	10·0
		8	30	41·87	,,	D	,,	H.A.	6·7
		9	25	24·04	264B Tauri	D	4·8	J.J.	10
		9	36	25·34	,,	D	6·7	J.J.	10

Date and G.M.T. of Observation.				Star and Phase of Occultation.		Mag. of Star.	Observer.	Aperture.
	d	h m	s			m		in.
1923 Jan.	27	9 36	25·16	264 B Tauri	D	6·7	H.A.	6·7
		12 35	51·19	Aldebaran	D	1·1	D.	10
		12 35	51·64	,,	D	,,	A.C.	3·7
Oct.	27	11 16	33·54	,,	D*	,,	A.C.	6·7
		11 16	33·89	,,	D*	,,	F.J.	10
		11 54	57·91	,,	R	,,	A.C.	6·7
1924 Apr.	8	8 47	12·31	Aldebaran	D	1·1	J.J.	3
		8 47	11·39	,,	D	,,	A.C.	3·7
		8 47	11·62	,,	D	,,	F.J.	10
		8 47	11·59	,,	D	,,	L.S.	6·7
Nov.	4	5 27	48·42	45 Capricorni	D	5·8	A.C.	6·7
	5	8 23	29·86	Mars 1L	R*	−0·8	A.C.	6·7
		8 23	30·24	,,	R*	,,	E.M.	12·7
		8 23	29·56	,,	R*	,,	P.M.	13
		8 24	57·61	Mars 2L	R*	,,	A.C.	6·7
		8 24	33·78	,,	R*	,,	P.M.	13
		8 24	53·32	,,	R*	,,	F.J.	10
		8 24	59·99	,,	R*	,,	E.M.	12·7
Dec.	7	5 7	48·44	ξ^2 Ceti	D	4·3	A.C.	6·7

Notes.

The asterisk indicates that the phenomenon occurred at the Moon's bright limb.
The dagger indicates that the limb of the Moon was eclipsed.
The first observation of reappearance, 1922 November 6, was made by tapping on the chronograph. The other by eye and ear.
The initials D., A.C., R.C., F.J., J.J., L.S., E.M., P.M., H.A., R.S., C.D., are those of Sir F. Dyson, Mr. Crommelin, Mr. Cullen, Mr. Jeffries, Mr. Jackson, Mr. Symms, Mr. Martin, Mr. Melotte, Mr. Acton, Mr. Smith, Mr. Davidson.
The names of the instruments are given with their aperture as follows:— Sheepshanks equatorial, 6·7 inches; altaz. portable telescope, 3·7 inches; photo-heliograph finder, 4·0 inches; Corbett, 6·5 inches; 26-inch equatorial, 26·0 inches; Merz equatorial, 12·7 inches; guiding telescope of the astrographic equatorial, 10·0 inches; detached telescope of 28-inch equatorial, 3·0 inches; astrographic equatorial, 13 inches.

The observations of the solar eclipse of 1921 April 7, the occultation of Venus 1921 July 1, and the transit of Mercury 1924 May 7, have already been communicated to the Society.

On the assumption that the error of the moon is wholly along its predicted track, each occultation gives a determination of the error of the moon in longitude. In forming the errors the places of the stars were revised from the latest available material, and the corrections to the Hansen-Newcomb longitude, due to Cowell, that are enumerated in *M.N.*, **83**, No. 6, were applied; also the values of Brown *minus* Hansen-Newcomb, as given in the same paper. (These values only include

terms of fairly long period.) The resulting mean values of Brown *minus* Observed for the different years are given, together with the means derived from the transit circle and altazimuth for the same years. The occultation of Mars was not used for this purpose.

Year.	Brown *minus* Observed Longitude.	
	Occultations.	T.C. and Altaz.
	″	″
1919	−8·52	−7·42
1920	−9·22	−7·96
1921	−8·75	−8·27
1922	−7·21	−8·50
1923	−7·10	−7·22
1924	−6·57	

As the number of observations of occultations each year is small, the results are only of small weight as compared with the transit circle.

Royal Observatory, Greenwich:
1925 *January* 7.

Micrometrical Measures of Double Stars (19*th Series*).

By Rev. T. E. Espin and W. Milburn.

In the following list, stars marked E were measured with the 24-inch reflector, and M with the 17¼-inch reflector.

Name.	R.A. (1900).	Decl.	P.	D.	Mags.	Nts.	Obs.	Date 1924.
	h m	° ′	°	″				
M 244	0 15·3	+63 52	274·6	3·06	10·0 13·0	2	M	·999
Aitken	15·6	64 9	288·1	2·65	9·5 9·8	2	M	·424
Espin 224	41·9	38 4	341·7	9·90	8·3 14·0	3	E	·671
h 1108	2 4·1	65 2	241·1	8·16	9·5 9·7	3	M	·584
Espin 270	10·4	36 0	358·3	2·78	9·6 11·8	2	E	·977 BC
Σ 349	3 2·6	63 28	323·3	6·62	..	2	M	·950
Espin 1718	4 57·3	40 5	272·7	1·99	9·3 9·4	3	E	·141
h 1156	5 17·2	70 13	224·5	7·47	11·0 12·7	4	M	·076
Σ 831	6 0·9	68 0	76·9	11·89	8·5 8·5	2	M	·114
β ——	7 17·6	37 57	64·0	1·96	9·5 9·6	3	E	·192
Σ 1079	17·9	38 0	331·0	6·07	8·5 9·0	3	E	·192
Σ 1122	36·4	65 24	4·3	15·55	..	4	M	·123
Σ 1139 rej.	42·5	37 20	14·2	16·11	9·3 10·5	2	E	·195
Espin 296	9 6·2	36 47	126·4	1·98	11·6 13·5	4	E	·284 BC
Σ 1334	12·6	37 14	238·7	2·45	5·0 8·0	2	E	·289
Σ 1339	14·7	37 9	69·1	1·73	8·3 8·7	2	E	·289

Name.	R.A. (1900).	Decl.	P.	D.	Mags.	Nts.	Obs.	Date 1924.
	h m	° ′	°	″				
Σ 1350	9 24·2	+67 14	248·3	10·60	7·0 7·2	3	M	·192
Espin 180	10 2·7	36 4	356·7	7·97	9·2 11·5	2	E	·289
Σ 1469	41·0	66 0	321·3	11·34	7·5 9·5	1	M	·290
Espin 183	18 4·5	36 41	165·7	8·91	8·8 11·0	2	E	·697
Ellison	51·1	36 48	339·7	8·72	11·0 11·3	3	E	·689 AB
			118·1	11·78	C=13·5	4	E	·696 AC
h 1369	19 4·3	36 45	321·4	8·62	9·5 9·7	2	E	·745
Σ 2473	5·1	37 45	289·9	5·96	9·0 9·1	2	E	·620
Holmes	9·9	38 58	332·0	11·83	10·5 11·5	2	E	·731
Σ 2508	14·4	67 44	118·7	18·19	..	2	M	·760
Holmes	16·9	34 55	254·3	10·82	10·5 11·0	2	E	·773
Holmes	44·6	36 55	292·9	10·05	10·7 11·5	2	E	·731
Holmes	51·9	34 27	76·4	10·95	9·5 10·0	2	E	·799 AB
			285·2	15·18	C=11·0	2	E	·799 AC
Holmes	53·8	34 38	270·3	7·41	9·3 9·5	2	E	·799
Holmes	55·3	35 23	232·0	5·74	11·0 11·2	3	E	·850 AB
			50·6	10·74	C=13·0	2	E	·842 AC
Holmes	20 7·6	35 21	121·2	5·66	9·3 9·6	2	E	·842
Σ 2747	58·4	37 15	259·3	4·73	8·0 8·2	4	E	·731
Holmes	59·2	36 21	261·5	11·80	7·5 11·0	3	E	·792
h 3044	21 31·4	71 7	78·0	7·47	9·0 9·1	2	M	·842
Espin 383	57·0	34 43	168·9	5·36	9·5 10·8	2	E	·799
Σ 2906	22 22·3	36 56	3·3	4·26	7·0 10·5	2	E	·953
h 1770	23·9	35 3	284·6	6·33	10·0 10·2	2	E	·745
Σ 2946	45·1	39 59	256·2	5·28	8·0 8·1	2	E	·953
Σ 2979	23 3·1	39 14	222·9	3·16	8·0 10·0	2	E	·953
Σ 3042	46·8	37 21	84·0	5·22	7·5 7·6	2	E	·977
A.G. 297	52·1	37 18	314·9	2·16	9·1 9·3	3	E	·999

Notes.

M 244. The earlier measures were made before the mirrors were resilvered, when it was a mere glimpse object. It is well seen now.

Aitken, $0^h\ 15^m{\cdot}6$. Found by Aitken (private letter) in looking for M 244. His measures 1923·83, P. 289°·6, D. 2″·44, 2 nights.

Espin 224 is B.D.+37°·133, not B.D.+37°·130. There is a 9·2 mag., P. 109°·4, D. 40″±.

Espin 1718 is B.D.+40°·1158, not B.D.+40°·1144.

Σ 1139 rej., Cl. IV., "Media trium in arcu." It is B.D.+37°·1777, not B.D. +37°·1778, as given in β's General Catalogue.

Ellison $18^h\ 51^m{\cdot}1$. A faint pair in field with δ^2 Lyræ. Star C was added with the 24-inch.

h 1369, *h* 123°·0, 126°·0. Either great change, or *h*'s mean should be 142°·7.

Holmes, $19^h\ 9^m{\cdot}9$. This, and the other doubles so named, are from a MS. Catalogue of Holmes' doubles.

Holmes, $19^h\ 16^m{\cdot}9$. A *comes* at P. 340°·5.
Holmes, $19^h\ 51^m{\cdot}9$. *Comes* Aa, P. 127°·5, D. 14″·16. AB is Scheiner 731. Scheiner by mistake makes Decl. +34° 37′. AC is Scheiner 730.
Holmes, $19^h\ 53^m{\cdot}8$ is Scheiner 745.
Holmes, $19^h\ 55^m{\cdot}3$ is Scheiner 759. C was added with the 24-inch.
Holmes, $20^h\ 7^m{\cdot}6$ is Scheiner 968 and 970.
Holmes, $20^h\ 59^m{\cdot}2$ is Scheiner 1380.

The following Double Stars have been found and measured with the 24-inch Calver reflector.

New Double Stars.

No.	B.D.	R.A. (1900).	Decl.	P.	D.	Mags.	Nts.	Date 1924.
	°	h m	° ′	°	″			
E 2079	+34·60	0 22·8	+35 12	344·8	4·62	9·3 12·0	2	·843
2080	+34·96	34·7	35 7	53·8	4·96	9·0 13·5	3	·878
2081	+34·97	35·3	34 59	48·5	2·74	9·2 11·0	2	·897
2082	+34·291	1 34·1	34 55	291·4	2·34	9·2 10·5	3	·878
2083	+37·633	2 41·0	38 10	335·5	6·10	9·1 11·2	2	·035
2084	+37·673	53·2	37 29	248·5	6·06	8·3 12·0	2	·977
2085	+37·867	3 57·2	37 43	263·8	3·88	7·7 9·8	2	·980
2086		4 14·0	38 22	202·8	3·28	10·0 10·1	2	·038
2087	+38·889	19·0	38 33	207·3	3·02	9·5 11·7	2	·111
2088		40·7	39 36	270·4	3·24	11·0 11·0	2	·111
2089		52·7	39 46	94·5	1·81	10·5 11·0	3	·078
2090	+40·1174	5 0·1	40 13	233·3	4·89	9·5 14·0	3	·179
2091		5·1	40 1	250·4	3·70	11·0 11·0	3	·179
2092	+39·1210	5·7	39 57	113·2	2·78	9·6 10·5	3	·179
2093	+39·1292	17·6	39 3	315·5	1·51	9·5 10·5	2	·196
2094		23·9	39 7	121·4	2·41	9·6 11·1	3	·195
2095	+39·1534	6 2·8	39 27	233·5	3·31	9·4 12·7	3	·144
2096	+39·1674	28·6	39 51	93·9	6·45	9·3 12·5	2	·178
2097	+39·1711	35·5	39 8	16·4	5·33	8·5 11·0	1	·194
2098	+39·1718	36·6	39 35	306·2	4·70	9·4 12·0	3	·188 BC
				199·9	28·24	A= 9·3	2	·186 AB
2099	+38·1753	7 18·2	38 20	280·4	4·14	9·5 13·0	2	·167
	+37·1729	23·7	36 59	292·8	11·32	8·4 12·0	1	·197
2100	+37·1739	26·5	37 47	190·3	2·91	9·3 9·8	2	·186
2101	+37·1771	40·5	37 25	4·9	5·29	9·2 9·3	3	·172
2102	+37·1797	47·8	37 2	268·9	6·35	9·2 12·2	2	·196
2103	+37·1811	52·0	37 16	180·7	3·08	9·4 10·5	3	·182
2104		8 30·4	36 18	158·9	2·83	9·8 12·0	2	·233
2105		47·9	37 11	102·1	2·10	9·5 10·5	2	·289
2106	+37·1954	9 6·9	36 58	337·2	4·27	9·4 10·5	2	·282
E 2107	+37·2017	37·2	36 53	170·2	2·23	9·5 10·5	3	·278

No.	B.D.	R.A. (1900). h m	Decl. ° ′	P. °	D. ″	Mags.	Nts.	Date 1924.
E 2108	+36·2025	9 59·3	+36 8	226·6	3·79	9·5 10·5	1	·290
2109	+37·3012	18 2·2	37 26	147·6	4·26	9·0 11·0	2	·607
2110	+36·3099	17·5	36 27	99·3	5·85	9·0 10·5	2	·735
2111		19 5·4	37 14	245·2	4·85	11·5 12·5	4	·768 BC
				174·1	5·48	A=10·5	3	·766 AB
2112	+37·3361	7·1	37 16	79·7	7·12	8·8 13·0	3	·704 AB
				344·7	9·80	C=11·5	3	·704 AC
2113		15·7	37 4	352·1	4·54	10·0 10·0	3	·710
2114	+37·3644	47·8	37 31	239·4	5·97	9·0 12·7	3	·629
2115	+37·3650	48·3	37 30	214·5	4·55	9·3 13·5	3	·658
2116	+36·3767	51·3	36 36	18·4	7·80	9·0 9·4	2	·818
2117		51·5	37 5	320·8	2·24	10·5 10·5	3	·704
2118	+37·3673	51·6	37 8	299·2	7·20	8·8 13·0	3	·704
2119	+37·3763	20 2·3	37 38	252·6	4·64	9·6 13·5	3	·798 BC
				257·1	12·32	A= 9·5	2	·764 AB
2120	+37·3816	8·1	37 15	286·0	5·95	8·2 12·7	2	·854
2121		9·9	38 10	233·5	3·10	9·2 10·6	2	·751
2122		39·4	37 27	63·6	4·72	9·6 11·5	3	·747
2123		48·0	37 57	99·6	4·39	8·7 11·5	4	·735
2124	+36·4536	21 19·0	36 21	268·9	6·45	9·5 11·0	2	·745
2125	+35·4513	20·4	36 6	45·0	2·20	9·4 10·5	2	·842
2126		20·6	36 5	172·4	4·23	11·0 11·2	2	·842
2127	+35·4535	23·7	36 10	298·2	4·43	9·8 10·0	2	·818
2128	+35·4542	25·4	36 9	287·5	7·14	9·0 13·5	4	·787
2129	+35·4602	35·8	35 38	310·0	5·02	10·0 10·1	2	·818
2130		36·5	35 45	121·8	3·18	10·5 12·0	3	·784
2131	+35·4609	37·7	35 45	221·6	6·41	9·3 10·5	2	·775
2132	+34·4719	22 29·9	35 11	215·8	2·18	9·0 11·0	2	·735
2133	+34·4726	31·4	34 48	68·8	3·87	9·0 12·5	2	·818
2134	+34·4828	59·2	34 38	113·7	1·45	9·3 11·5	2	·735
2135	+35·4971	23 6·8	35 16	207·9	2·37	9·3 9·5	3	·569
2136	+34·4922	20·4	34 55	357·7	3·37	9·3 11·0	3	·880
E 2137	+34·4987	36·7	35 9	138·2	6·50	9·5 11·5	3	·569

Notes.

2083. Distant *comes* at P. 284°·8.
2091. This is a distant *comes* to B.D.+39°·1203 at P. 143°·4.
2094. At P. 241°·7, from B.D.+39°·1324.
2101. *Comes* to B., 13 mag., P. 50°·2, D. 16″·7 single setting.
2109. Distant *comes*, 13 mag. N.
2123. The south star of a wide pair. The other star mag. 8·6 is at P. 327°·8, D. 60″±. Neither star is in the B.D.
2128. Distant *comes* F.

Stars Missing or Faint.

	h m s	° ′	
B.D. +39·996	4 18 11·9	+39 10·5	mag. 9·5 (1855) 1924 Feb. 15. Not above 11·5.
+39·2553	12 37 1·0	+39 10·5	mag. 9·5 1924 May 5, May 15. Not certainly seen.
+39·2567	12 42 44·8	39 54·4	mag. 9·5 1924 May 15. Mag. 12·0
+34·4685	22 20 28·2	34 48·2	mag. 9·5 1924 Sept. 24. No star seen here.

The following Double Stars have been found and measured with the 17¼-inch Calver reflector.

No.	B.D.	R.A. (1900).	Decl.	P.	D.	Mags.	Nts.	Date 1924.
	°	h m	° ′	°	″			
M 323		0 10·0	+66 26	146·2	2·09	9·5 10·0	3	·883
324	+65·45	17·4	65 50	149·2	6·41	9·5 11·7	3	·778
	+65·49	19·1	65 43	239·1	10·57	9·5 9·5	2	·797
325		26·1	67 2	343·5	3·86	9·5 11·5	2	·982
326		26·7	67 4	140·0	3·33	10·0 10·5	2	·977
327		32·6	66 32	69·5	6·24	9·5 11·5	2	·982
328		1 22·1	62 47	111·9	2·91	9·7 12·0	2	·900
329		22·8	62 46	351·2	3·12	12·0 12·2	3	·564 AB
				154·7	11·76	C=13·0	3	·564 AC
330		22·9	62 47	327·2	2·15	11·0 11·7	3	·564 AB
				329·7	8·52	C=12·7	3	·564 AC
331		47·9	61 38	144·4	4·90	11·5 12·5	2	·982
332	+62·418	2 26·9	62 30	271·9	1·99	9·5 10·7	3	·328
333	+64·377	3 6·2	64 11	310·4	4·96	9·5 10·0	2	·453
334		33·2	61 53	351·4	3·64	10·0 13·0	4	·511
335	+61·629	34·8	61 33	70·7	4·10	10·0 10·1	3	·967 BC
				257·2	36·62	A= 9·5	2	·955 AC
336		48·9	64 30	176·2	3·02	10·0 11·0	4	·978
337		5 17·3	65 31	153·6	5·44	9·5 10·0	1	·953
338	+65·475	20·7	65 30	201·8	6·05	9·5 12·2	1	·953
339	+65·486	32·9	65 5	56·4	9·21	8·5 10·0	3	·099
340	+64·548	40·7	64 54	168·5	5·63	9·5 10·0	2	·178
341		6 19·1	65 15	233·7	5·56	10·5 10·7	2	·119
342		22·0	65 0	355·8	5·75	10·0 10·2	3	·103
343	+66·455	29·4	66 15	158·2	2·58	8·0 12·5	2	·193
	+66·468	40·4	66 2	209·5	12·17	9·0 10·0	1	·192
344		7 49·6	66 4	253·3	4·70	9·8 9·9	2	·167
345		8 40·1	66 54	265·4	3·86	10·0 10·1	3	·182
346		56·0	66 44	303·2	3·13	10·0 10·1	3	·189
347	+66·639	9 47·3	66 37	59·8	7·23	9·2 10·7	3	·182
M 348		10 45·2	64 58	354·6	3·31	10·0 10·0	4	·283

No.	B.D.	R.A. (1900).	Decl.	P.	D.	Mags.	Nts.	Date 1924.
	°	h m	° ′	°	″			
M 349		10 49·7	+64 50	138·5	6·82	10·0 10·5	2	·289
350		12 57·4	63 3	71·0	4·44	11·0 12·5	1	·342
351	+63·1048	13 1·1	63 19	153·3	2·52	8·0 12·5	2	·353
352		14 20·7	64 23	313·3	4·98	10·0 10·5	1	·391
353		24·8	63 28	107·7	3·37	10·2 10·6	2	·382
354		15 17·4	63 29	120·5	3·14	10·0 11·0	1	·391
355		18 41·4	68 4	223·7	7·23	11·0 11·5	3	·611
	+67·1119	19 7·9	67 46	43·9	12·48	8·5 10·7	2	·645
356		14·6	67 22	26·6	3·29	9·8 10·5	2	·678
	+67·1187	41·6	67 18	181·7	11·41	8·0 10·0	2	·620
357	+68·1081	45·7	68 53	259·5	1·72	9·5 10·4	2	·735
358		50·3	68 42	244·7	3·64	11·0 13·0	4	·771
359		20 0·1	67 42	284·0	4·21	10·0 10·5	2	·727
360		16·0	69 54	215·2	2·56	9·5 9·6	3	·689
361		22·3	68 13	323·5	2·76	9·7 9·8	2	·620
362		28·2	69 55	34·0	4·33	10·0 11·0	2	·842
363		46·1	70 14	218·5	2·87	9·5 12·5	2	·813
364		21 13·7	69 38	63·5	3·98	9·7 11·7	2	·678
365	+69·1161	19·6	69 18	52·2	4·32	9·5 10·5	2	·745
366		31·2	69 42	155·8	3·22	10·0 11·2	3	·764
367		31·8	70 32	111·8	3·45	10·5 10·5	1	·843
368		34·8	69 58	77·6	3·37	11·5 11·6	3	·704
369		22 2·3	67 6	109·3	5·67	11·0 13·0	3	·727
370		7·6	67 41	69·2	3·12	9·5 9·5	3	·885
		7·7	67 42	359·0	11·26	9·3 11·0	2	·885
371		43·1	55 20	56·4	3·67	10·0 11·0	2	·943
372		48·6	56 44	321·1	5·52	10·7 10·7	3	·767
373	+56·2891	48·9	56 46	343·5	6·54	9·0 11·5	3	·767
374		52·1	56 25	76·6	4·59	10·0 10·0	3	·740
375		53·8	56 17	16·1	2·30	9·5 10·0	4	·840
M 376		53·9	56 15	239·9	5·98	9·7 10·1	3	·801

Notes.

329, 330 are in the cluster N.G.C. 559.

335. There is a *comes* of 10·2 mag. at 190°·4, 120″±, and another of 10·0 mag. at 75°·0, 220″±, from A.

340. There is a star of 10·0 mag. at 172°±, 35″±.

355. There is a *comes* of 12·0 mag. preceding A and a star of 8·7 at 150°·2, 90″–100″±.

359. There is a star of 9·7 at 331°·9, 35″±.

Observations of Mars in 1924. By C. Luplau Janssen, M.Sc.
(Plate 12.)

During the summer of 1924 the planet Mars was observed here at the Urania Observatory, Copenhagen. The instrument used was the 246 mm. refractor equipped with eye-pieces giving powers varying from 209 to 469 diameters. In advance no results were expected, in view of the very unfavourable position of the planet, viz. 10°–20° only above our horizon. Fortunately the weather was unusually fine here this summer, often the seeing was very good, and consequently it was possible to examine the Martian details very closely. A number of drawings, some of which are shown here, have been made, and a small number of micrometrical measures have been secured. These will be published later. In the following I shall give a short summary of our observations. The observers were C. Luplau Janssen (J.) and Svend Lauritzen (L.). The times are G.M.T.

June $20^d\ 12^h\ 25^m$ (J.). Seeing poor. Only the white polar regions can be distinguished.

July $12^d\ 12^h$ (J. and L.). Seeing very good. All the details seen are shown on the drawing.

„ $15^d\ 12^h$ (J. and L.). *Fastigium Aryn* is easily seen. The dark gap in the southern polar cap has grown more distinct since the previous observation.

„ $24^d\ 11^h\ 50^m$ (J.). Seeing not very good. The *Syrtis Major* region is visible. Some streaks are seen. *Hellas* is nebulous. *Solis Pons* and *Lunæ Pons* are rather distinct.

„ $27^d\ 11^h\ 30^m$ (L. and J., Böye). *Syrtis Major* and *Syrtis Minor* are very distinct. *Hellas* is pale. *M. Hadriacum* is dark. *Hesperia* rather bright. *Nepenthes*, *Triton*, and *Amenthes* are seen to the left of *Syrtis Major*. At right *Astusapes* is seen with difficulty.

„ $29^d\ 11^h$. Seeing very poor. No details are visible.

Aug. $1^d\ 12^h$ (J. and L.). Seeing is very good. *Titanum Sinus* is very dark. A very great number of streaks are seen at the first glance. *Elysium* is white. *Hesperia* is bright. The islands of *Mare Australe* are very indistinct. The border of the south polar cap is very dark.

„ $2^d\ 12^h$ (L.). Seeing very poor.

„ 8^d (L. and J., Böye, Fjeltofte). Seeing very bad. It is impossible to observe anything.

„ 9^d (J. and L., Böye, Fjeltofte). Very good images. *Nodus Gordii* is very dark, almost black. *Mare Sirenum* is greyish, and the islands are very diffuse. Many streaks are seen.

„ $10^d\ 10^h\ 30^m$ (J. and L.). Seeing excellent. *Nectar* is very broad and very dark. *Agathodæmon* is as dark as the seas. No important white regions are remarked. The whole disc seems to be veiled by grey diffuse masses which grow brighter towards the edge of the disc.

Aug. 12^{d} 11^{h} (L. and J.). Seeing rather poor. The appearance of the planet has not changed since August 10.

„ 13^{d} (L. and J., Böye). Images excellent. *Lacus Solis* is very dark and *Nectar* is as broad as *Lacus Solis* itself. *Ophir* is brilliantly white. *Agathodæmon* is very marked, and the many streaks seen are very distinct. *Mare Australe* is dark. *Margaritifer Sinus* and *Auroræ Sinus* are as black as ink. The edges of *Ganges* are very diffuse. The islands are very pale. *Lacus Niliacus*, on the edge of the disc, is astonishingly dark and distinct.

„ 16^{d} 11^{h} (L. and J.). Seeing only good for short intervals. The polar cap is irregular. On the edge of it a brilliant white spot is seen. Otherwise the appearance has not changed since the last night of observation.

„ 17^{d} 11^{h} (J.). Seeing rather poor. Dark streaks are seen in *Auroræ Sinus*. *Thyle* is white (on the edge of the disc).

„ 19^{d} 9^{h}–12^{h} (J.). Many streaks are seen. Seeing good.

„ 20^{d} 10^{h} (J.). No remarks.

„ 21^{d} 10^{h} (J.). No new details are seen. The polar cap has become rather small.

„ 24^{d} 10^{h} (J. and L., Böye). Seeing very good. On the border of the polar cap numerous small brilliant white spots are seen. *Sinus Sabæus* and *Furca* are very dark. At the upper edge at right a brilliant white region. *Hellas* is red, and the coloration of it is very pronounced. A dark vertical streak divides *Deucalionis Regio* into two parts. The streaks are not very numerous here.

„ 25^{d} 9^{h}–12^{h} (J., Böye). *Sinus Sabæus* is very dark. All the well-known main streaks of this region are easily seen. The polar cap is now very small, with two very brilliant white points. Seeing excellent.

„ 26^{d} 11^{h}. Seeing very good. The polar cap is now very small. No special remarks are made.

„ 27^{d} 11^{h} (J. and L., Fjeltofte). No special remarks are made. The appearance of the planet has not changed since the previous night. Many streaks are registered.

„ 27^{d} 13^{h}. *Xisuthri Regio* is now seen very distinctly divided from the continent by a very dark streak.

Sept. 3^{d} 11^{h} (J.). Seeing very bad.

„ 4^{d} 8^{h} (J.). Seeing not good. The polar cap has become very small. *Mare Acidalium* is very dark. *Jamuna* has become broad. *Auroræ Sinus* is black.

„ 6^{d} and 7^{d}. Seeing very bad.

„ 10^{d} 9^{h} 30^{m} (J.).

I. *Mare Sirenum* and *Mare Kimmerium* are very dark. *Trivium Charontis* very dark. *Cerberus* very broad and dark. *Lœstrygon*, *Eumenides*, and *Tartarus* are easily seen. Mist in the air prevents a closer examination.

II. 10^{h} 30^{m}. The "seas" are rather pale, but the southern coasts are well defined. The colour of the continent is

strong red. The streaks are easily seen, and are decidedly continuous. *Zephyria* is white at the coast. At the end of the observation the seeing becomes poor.

Sept. 11^{d} 8^{h} 45^{m} (J.). Seeing very poor, but the dark seas are seen.

,, 13^{d} 8^{h} 10^{m} (J. and L.). Seeing poor. At the lower edge of the disc a dark region is seen. The polar cap is very small, but still visible.

,, 14^{d} 9^{h} (J. and L.). Very bad seeing.

,, 15^{d} 9^{h} (J. and L.). Seeing very poor.

,, 16^{d} 9^{h} (J. and L.). The polar cap only is visible.

,, 18^{d} 10^{h} (J. and L., Fjeltofte). Seeing rather good, but no special remarks are made. A brilliant white spot is seen at the border of the polar cap.

,, 27^{d} 10^{h} (J. and L.). Seeing rather poor. On the right edge of the disc *Hellas* is bright. The polar cap is bright and still easily seen.

,, 28^{d} 9^{h} (J. and L.). Seeing bad. *Gehon* and *Hiddekel* are seen.

,, 29^{d} 9^{h} (L. and J.). Seeing rather good. Many details are easily seen. *Deucalionis Regio* is very broad. *Pyrrhæ Regio* is diffuse. The southern sea—*Mare Australe*—is very dark. *Furca* and *Sinus Sabæus* are quite black. *Margaritifer Sinus* is also very dark. *Gehon*, *Hiddekel*, *Phison*, *Euphrates*, and *Typhon* are easily seen. A bright bridge exists connecting *Hammonis Cornu* and the extreme regions of *Deucalionis Regio*.

After this date the seeing was constantly too poor for observation. The observations have accordingly been discontinued.

Drawings.

		G.M.T. h m		
1	Aug. 1	12 0	Made by C. Luplau Janssen.	
2	,, 10	11 45	,,	,,
3	,, 13	11 0	,,	,,
4	,, 16	11 0	,,	,,
5	,, 27	10 0	,,	,,
6	,, 27	12 0	Made by Svend Lauritzen.	

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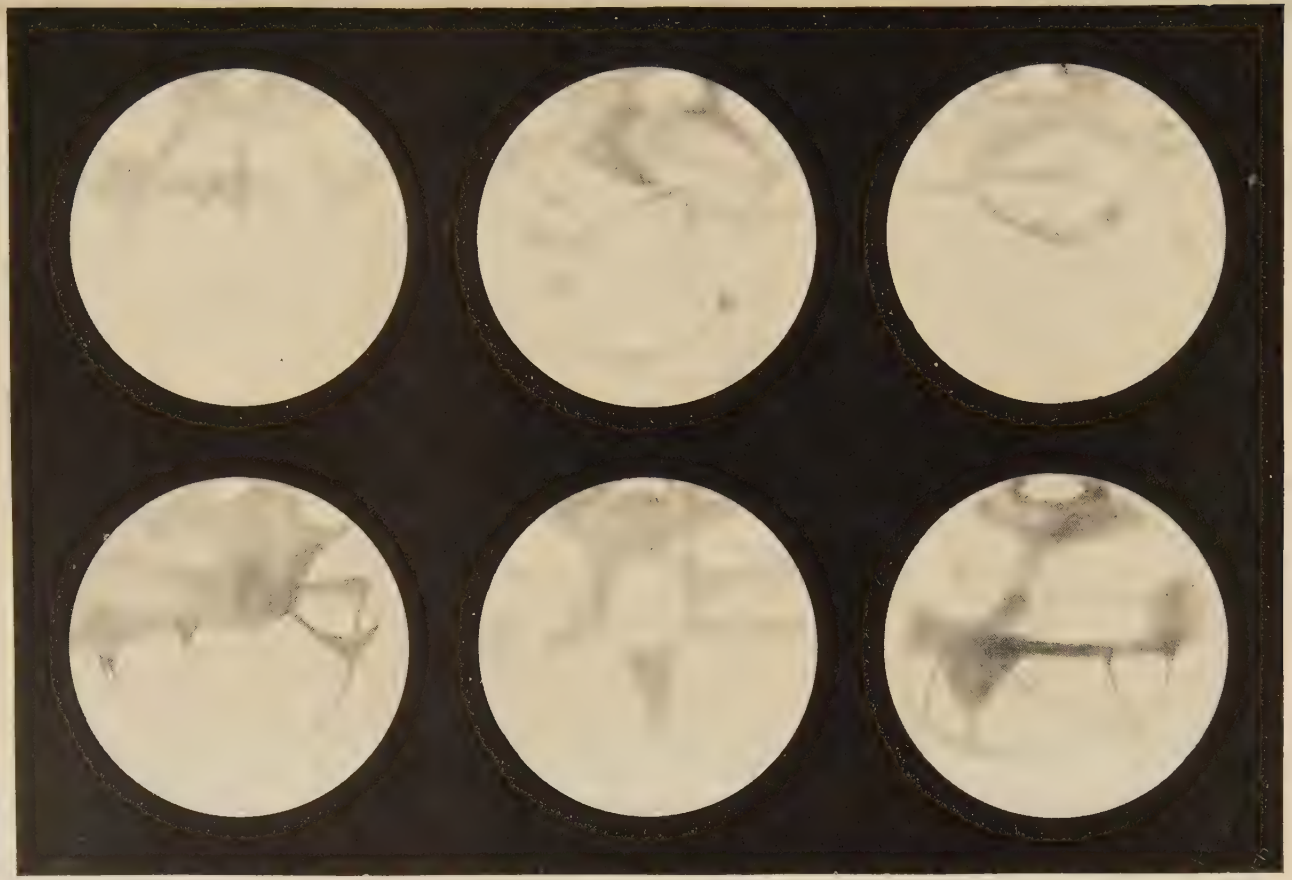


Aug. $16^{d}\ 11^{h}\ 0^{m}$ G.M.T.
$\lambda = 60°$ $\phi = -17°$

Aug. $27^{d}\ 10^{h}\ 0^{m}$ G.M.T.
$\lambda = 308°$ $\phi = -16°$

Aug. $27^{d}\ 12^{h}\ 0^{m}$ G.M.T.
$\lambda = 338°$ $\phi = -16°$

The Determination of the Curvature Radius of Space-time. In Reply to Dr. Knut Lundmark. By Ludwik Silberstein, Ph.D.

Dr. Lundmark's valuable paper on this subject, published in *M.N.*, **84**, 747–770, 1924, calls for some remarks. But before proceeding to these I should like to express my deep gratitude to Dr. Lundmark for having devoted so much attention to the discussion of this problem from a perfectly impartial attitude.

The globular clusters, which in my opinion are most promising, do not seem to Dr. Lundmark to afford proper material for a determination of the curvature radius "with any accuracy, at least for the present." He turns, therefore, to examine other classes of distant (though on the whole not sufficiently distant) celestial objects, viz. the Cepheids, Novæ, O-stars, eclipsing variables, R-stars, N-stars, and the spirals. It is this extension of the field of attack which seems particularly welcome, and wherein Lundmark makes a valuable contribution to the subject under investigation. Some of his results, as notably those derived from O-stars, R-stars (*M.N.*, **84**, 761, 762), and spiral nebulæ (pp. 765–6), are even "rather favourable" to my formula, and yield R-values fairly close to that originally computed by me, which was $6 . 10^{12}$ astr. units.* Yet, as the distances of most of these objects, with the exception of the spirals (whose distances are not known), are much too small to show the effect of a finite R, the globular clusters still appear to me by far the best material at present available and most worthy of further extension.† Lundmark turns away from the globular clusters, and the two Magellanic clouds (Table I.), because they give a dispersion in R too large to be explained by the uncertainty in the relative parallaxes of the clusters. This, however, is due to the fact that he computes the R-values by means of

$$D = \frac{\delta\lambda}{\lambda} = \pm\frac{r}{R} . \quad . \quad . \quad . \quad . \quad (A)$$

which I have used only provisionally, for a narrower group, for the sake of a first rough approximation, and which is only the first term of the formulæ

$$D^2 = \frac{r^2}{R^2} + \frac{{v_0}^2}{c^2}$$

for radial motions passing through the origin (the sun), and

$$D^2 = \frac{r^2}{R^2} - \frac{{r_0}^2}{R^2}$$

for such as are reversed at a finite minimum distance r_0 (perihelion). Even these are only special cases of the general formula holding for any, not only radial, inertial motion, with v_0 written for the velocity at the perihelion,

$$D^2 = \left(1 - \frac{{r_0}^2}{r^2}\right)\left(\frac{r^2}{R^2} + \frac{{v_0}^2}{c^2}\right) \quad . \quad . \quad . \quad . \quad (1)$$

* By a slip the R-values quoted by Lundmark are all printed as "km." instead of "astr. units."

† Of the 86 globular clusters of known distance, there are still 68 awaiting the determination of their radial velocities.

which I have since published (*Nature* for Sept. 6, and *Phil. Mag.*, **48**, 619) and used throughout as a working formula.* Needless to say, radial motions which were assumed only for the sake of mathematical simplicity will not cover, even approximately, the actual case of one in many thousand celestial objects, and the formula to be used is (1), holding for any oblique inertial motion as a first-order approximation of the rigorous formula (8) (*Phil. Mag.*, *loc. cit.*). If, instead of this, the truncated formula (A) is used for evaluating the radius R from single objects, an enormous dispersion in R is bound to occur, owing to the deviations of the star-constants v_0^2 from a mean value $\overline{v_0^2}$, and, still more, of the first factor in (1) which contains the other star-constant r_0 and which, for any given r, has a range from 1 to zero. In accordance with this formula an object, no matter how distant, will show a small spectrum shift if it is near its perihelion, without in the least calling for an infinite or an abnormally high R-value. In other words, an object which moves transversely will show no effect,† the transverse Doppler effect, of second order and much too small to be ever observed, being already neglected in passing from the rigorous formula to (1).

This will account in a natural way for the large R-dispersion in Dr. Lundmark's table, and, above all, will rule out his $R=\infty$ for N.G.C. 6626, and also his $96 \cdot 10^{12}$ and $77 \cdot 10^{12}$ for N.G.C. 7089 and 5904. These clusters are just passing through their perihelia or nearly so. Of course, such a situation is not likely to happen often. Actually it happens only once completely and thrice approximately in twenty cases. Similarly for the few remaining objects showing low D's at large distances. This settles the objection based on large dispersion.

At any rate, however, formula (1) requires that, in spite of all possible variations, $|D|$ should *on the whole increase with r*, and therefore calls for a positive correlation coefficient between the two measured variables. It thus is of paramount importance to compute this coefficient. Having done so on several occasions, when I had at my disposal only 10, then 14, and finally 21 distant objects, I found rather high values ‡ for the Bravais-Pearson coefficient, and, up to 18 objects, increasingly high and encouraging ones. Dr. Lundmark finds (p. 752) for his table of 20 objects a very low correlation coefficient, $0{\cdot}255 \pm 0{\cdot}227$, which barely exceeds its P.E., and which would thus testify of practically no correlation at all (or $R=\infty$). I assume that he used in this computation Shapley's distances, which are also those used in my table. But Dr.

* This formula, of course, did not come to Dr. Lundmark's knowledge, as it had not been published at the time of his writing.

† This, of course, is no special feature of the de Sitterian world or of relativistic physics. It is perfectly familiar from classical physics. Notice that in the latter case, when $R=\infty$, formula (1) reduces at once to

$$c^2D^2=v_r^2=(1-r_0^2/r^2)v_0^2,$$

and since at the same time the inertial motion is rectilinear and uniform, this formula simply reads: radial velocity = projection of resultant constant velocity upon the line of sight.

‡ One of these, for 14 objects, amounting to 0·712 and exceeding eight times its P.E., is given on p. 552 of *Theory of Relativity*, 2nd edition; another, still higher one, for 18 objects, given at the recent Toronto meetings, will be quoted presently.

Lundmark takes the radial velocity of N.G.C. 1904 to be +300 instead of +200, the value obtained privately from Professor V. M. Slipher last August together with his whole list of clusters. This makes a considerable difference. Another, though lesser, effect upon the correlation estimate is due to −190 as radial velocity of N.G.C. 5024, which, according to Slipher, is only −170. (Similarly, −135 instead of Slipher's −125 for N.G.C. 5272; but this scarcely affects the coefficient.)

Under these circumstances I may be permitted to quote here in full my own table of these 20 objects, with the addition of one spiral, and the corresponding diagram, as shown on slides in reading two papers at the meetings of the British Association and the International Mathematical Congress, 1924 August, at Toronto, only the data for the Lesser Magellanic Cloud being here modified in accordance with Shapley's last estimate (*Harvard Coll. Obs.*, *Circular* No. 255). The distance estimate of the spiral Messier 33 (N.G.C. 598, or 604?) is due to Dr. E. Hubble. The 21 objects are numbered through merely for the sake of easier reference. Their N.G.C. numbers are given in the second column. The distances r are in kiloparsecs.

Distances and Radial Velocities of 18 *Globular Clusters, the Magellanic Clouds, and* 1 *Spiral.*

	Object.	r.	cD.		Object.	r.	cD.
(1)*	6205	11·1	−300	(12)	6626	18·5	zero
(2)	6341	12·3	−160	(13)	5024	18·9	−170
(3)	6218	12·4	+160	(14)	6093	20·0	+70
(4)	5904	12·5	+10	(15)	6333	25·0	+225
(5)	5272	13·9	−125	(16)	L.M.Cl.	31·0	+168
(6)	7078	14·7	−95	(17)	1904	25·6	+200
(7)	6266	15·2	−50	(18)	598 (Messier 33)	31·3	−260
(8)	7089	15·6	−10				
(9)	6273	15·9	+30	(19)	6934	33·3	−350
(10)	7099	17·2	−125	(20)	G.M.Cl.	35·0	+276
(11)*	1851	17·2	+300	(21)*	6229	43·5	−100

Let us first take all the 21 objects. The arithmetical means of the data are, to three figures,

$$\bar{r} = 21{\cdot}0, \qquad c\,|\overline{D}| = 152,$$

whence the deviations $x = r - \bar{r}$, $y = c\,|D| - c\,|\overline{D}|$. These give

$$\Sigma xy = +6986, \qquad \Sigma x^2 = 1638, \qquad \Sigma y^2 = 2212 \cdot 10^2,$$

whence the correlation coefficient $k = \Sigma xy / \sqrt{\Sigma x^2 \cdot \Sigma y^2} = 0{\cdot}367$, and its probable error, $Q = 0{\cdot}675 \frac{1 - k^2}{\sqrt{21}} = 0{\cdot}127$. Thus, for the 21 objects,

$$k_{21} = 0{\cdot}367 \pm 0{\cdot}127 \quad . \quad . \quad . \quad . \quad (2)$$

which, though considerably greater than Dr. Lundmark's, is yet a poor correlation coefficient, only about three times its P.E.

In the next place, however, let us discard from the computation of k the three objects (1), (11), and (21), marked with asterisks, without doubting their accuracy,* but merely because of their eccentric position as seen from the accompanying diagram. Then, for the remaining 18 objects,

$$\bar{r} = 20^{\cdot}5, \qquad c\,|\overline{D}| = 138,$$
$$\Sigma xy = +9862, \qquad \Sigma x^2 = 1013, \qquad \Sigma y^2 = 1721 \cdot 10^2,$$

whence the correlation coefficient

$$k_{18} = 0^{\cdot}747 \pm 0^{\cdot}070 \quad . \quad . \quad . \quad . \quad (3)$$

i.e. about 10·7 times its probable error. This testifies to a very strong correlation of the kind required by the theory, and the discarding of 3 out of 21 objects is certainly a legitimate procedure (without prejudice to any of the data), seeing that it doubles k and almost halves its probable error. This result vindicates the quality of this group of objects as bearing a strong evidence for a finite curvature radius. Of course, a score or more data (preferably out of the 68 globular clusters) would be very desirable. Should the accession of these new data lower considerably the value of k/Q, this evidence will be invalidated; but if, on the contrary, the latter ratio be thereby increased or even only kept at its present level, the finite curvature radius will be firmly established.

Next, as to its numerical value. Since the two star-constants r_0, v_0 appearing in formula (1) are unknown and, especially for these distant objects, remain inaccessible to measurement,† R can be determined only statistically. Now, if all the inertial orbits were, roughly at least,

* Though of the radial velocity of (21) Professor Slipher actually wrote me that it is "less accurate than the others."

† If in addition to the radial velocity the proper motion and thence the transverse velocity v_t of an object is accessible to measurement, the two star-constants $r_0 = R\sigma_0$, $v_0 = c\beta_0$ can be reduced to a single additive constant $A = \beta_0^2 - \sigma_0^2$. In fact, the resultant velocity $v = \sqrt{v_r^2 + v_t^2}$ in any inertial motion is given rigorously by

$$\frac{v^2}{c^2} = \cos^2\sigma[1 - \cos^2\sigma(\sec^2\sigma_0 - \beta_0^2)], \quad \sigma = r/R,$$

and, to our degree of approximation, by

$$\frac{v^2}{c^2} = \frac{v_0^2}{c^2} + \frac{1}{R^2}(r^2 - r_0) = \frac{r^2}{R^2} + A.$$

This formula may offer a practical advantage, as it is free from the unknown factor $1 - r_0^2/r^2$ having the enormous range of uncertainty 1 to 0.

It may be well to add that the transverse velocity itself is, rigorously,

$$v_t = v_0 \frac{\cos^2\sigma/\sin\sigma}{\cos^2\sigma_0/\sin\sigma_0},$$

and, approximately,

$$v_t = v_0 \frac{\sigma_0}{\sigma} = v_0 \frac{r_0}{r},$$

the latter in spite of the circumstance that the orbit need not be even roughly rectilinear.

rectilinear, the factor $1 - r_0^2/r^2$ would, in spite of the ellipticity of space and in view of the smallness of r/R, be simply the squared cosine of the inclination of the resultant velocity to the line of sight, and therefore, if all angles of inclination be assumed equally likely, the average value of this factor would be $\frac{1}{3}$. Since, however, the orbits are hyperbolically shaped curves which, even for small $\sigma_0 = r_0/R$, may form more or less slender loops, if $\beta_0 = v_0/c$ be correspondingly small,* that factor has no longer this simple meaning. It seems reasonable instead to assume, provisionally at least, that for any given r all values of r_0/r from 0 to 1 are equally likely. Then the mean value of the first factor in (1) will be $\frac{2}{3}$, and the statistical curve, along which the points $r, |D|$ in a correlation graph may be expected to concentrate, will have the equation

$$D^2 = \tfrac{2}{3}\left(\frac{r^2}{R^2} + \frac{\overline{v_0^2}}{c^2}\right) \quad . \quad . \quad . \quad . \quad (4)$$

The mean $\overline{v_0^2}$ can either be derived from the group of objects under investigation (by splitting it into two sub-groups ; *loc. cit.*, p. 626) or else borrowed from the majority of the nearer stars, say $\sqrt{\overline{v_0^2}} = 30$ km./sec., to serve as a general mean for all objects. The values of the constants v_0 (though not of v_r) and their distribution among actually distant objects need not differ from those for the nearer stars, and the actual orbital velocities of the latter cannot possibly differ much from their own minimum velocities v_0, since the distance effect of a few or even a hundred parsecs is but small.

Such a curve (4), with $(\overline{v_0^2})^{\frac{1}{2}} = 30$, is drawn across the accompanying diagram, where the labelled circles represent the 21 objects, their centres being placed in accordance with the pairs of data of the preceding table. The value of the curvature radius appearing in the equation of this curve is

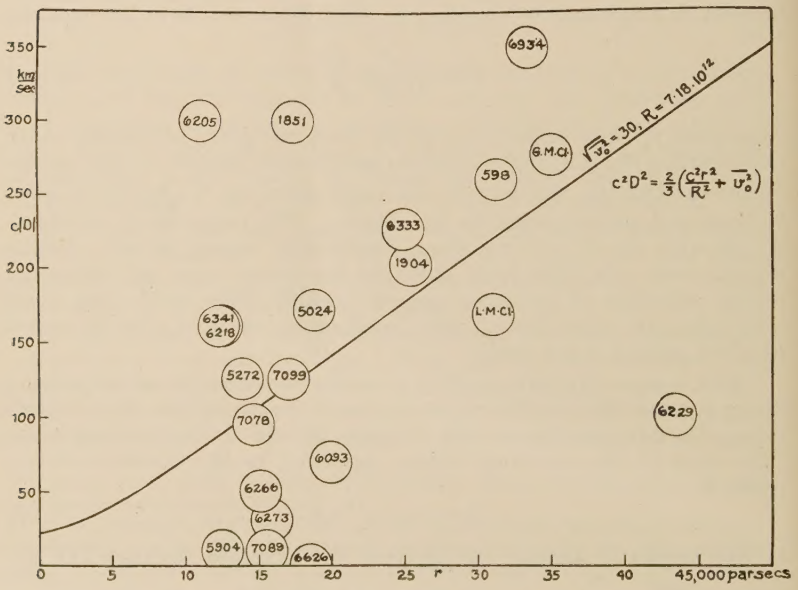
$$R = 7{\cdot}2 \,.\, 10^{12} \text{ a.u.} \quad . \quad . \quad . \quad . \quad (5)$$

It was originally derived by applying the averaged equation (4), *i.e.* $\frac{c^2}{R^2}\overline{r^2} = \frac{3}{2}c^2\overline{D^2} - 900$, to 18 objects only, namely, those of the table less N.G.C. 6205, discarded in view of its exceptionally high v_0, and less N.G.C. 1851, 1904, whose radial velocities were not then known to me. It was encouraging to see that the latter of the two, when their data came, fell so close to the curve, without calling for a revision of the R-value. The deviation, in fact, is so small that a recalculation of R would scarcely affect the first decimal figure. As will be seen from the diagram, there is a fairly marked tendency to follow this curve, nine or possibly eleven of the circles arraying themselves rather closely along it. Future data of this kind may, of course, either weaken or strengthen the rôle of this curve. For the present, however, $7{\cdot}2 \,.\, 10^{12}$ would seem about the best value obtainable for the curvature radius.

The scattering, on both sides of the curve, is amply provided for by

* The semi-amplitude θ_p of any such orbit is determined by $\tan\theta_p = \frac{\beta_0}{\sin\sigma_0}$. Cf. *Phil. Mag.*, *loc. cit.*, especially p. 623.

equation (1), valid for each object separately, without in the least questioning the accuracy of the observational data. An object like N.G.C. 6205 will be far from its perihelion and will have its v_0 much higher than 30, a fact which holds for some of the nearer stars also. The objects situated low down, such as N.G.C. 6626, have only to be placed very near their perihelia to be explained. It may be noted in this connection that, in accordance with the formula $\tan\theta_p = \beta_0/\sin\sigma_0$ mentioned before, the larger r_0 or σ_0, and therefore also the actual r, the more slender the quasi-hyperbolic orbit, so that a comparatively small excursion from the perihelion will bring about an observable radial



velocity. Whence it follows that low or evanescent values of D are less likely for the more distant objects. Even this theoretical feature is, thus far, borne out by the correlation graph.

Some of the groups of comparatively near objects discussed by Dr. Lundmark would fit well enough into this diagram. Thus the representative point of the O-stars, with 1·6 kiloparsecs and 24 km./sec. as means (Lundmark, p. 761), would be placed almost on the curve itself, and similarly the points representing two small groups of R-stars, placed at 1·7 and 2·6 kiloparsecs, with a radial velocity 34 (p. 762). Dr. Lundmark's hypothetical distance estimate, 36 kiloparsecs, for the Andromeda nebula (which, however, is partly based on the theory at issue) would place this object just above and slightly to the right of "G.M.Cl." and thus close enough to the curve.

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